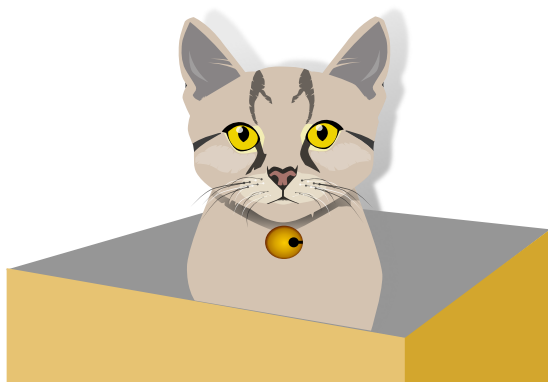


QUANTUM MECHANICS IN YOUR FACE FOR ANYBODY

QUANTUM MYSTERIES IN YOUR FACE FOR ANYONE



ANDREW FOWLIE

MTHo20. MODELLING THE REAL WORLD

CONTENTS

1. Mermin Experiment Setup
2. Mermin Experiment Results
3. Hidden Instruction Model
4. Bell's Theorem
5. Welcome to the Quantum World

THE QUANTUM WORLD

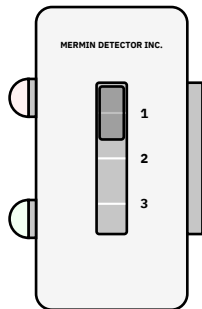
- ▶ Big, everyday objects behave in predictable and intuitive ways
- ▶ Small things — particles and atoms — obey a different set of rules, governed by *quantum mechanics*
- ▶ The quantum world is strange and counter-intuitive
- ▶ Mermin (1981) wanted everyone to see that strangeness without requiring any mathematical or physical concepts
- ▶ He created an experiment to show the difference between quantum physics and everyday life

Section 1

MERMIN EXPERIMENT SETUP

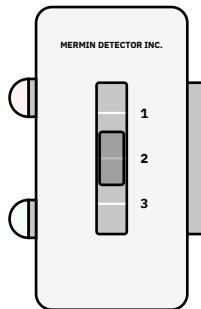
MERMIN DETECTOR

- ▶ This device is a *Mermin detector*
- ▶ It detects particles that strike its right-hand side
- ▶ It has a switch that can be in position **1**, **2** or **3**
- ▶ When it is struck by a particle, either the red light or the green light flashes



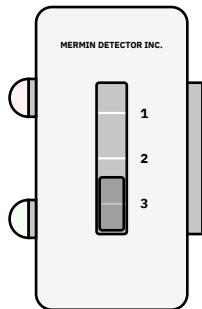
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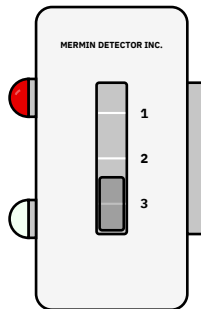
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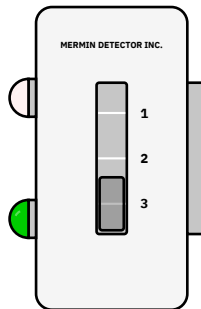
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- ▶ When it is struck by a particle, either the **red light** or the green light flashes



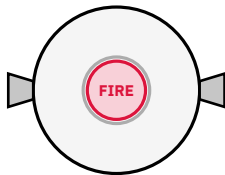
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- ▶ It has a switch that can be in position **1**, **2** or **3**
- ▶ When it is struck by a particle, either the red light or the **green light** flashes



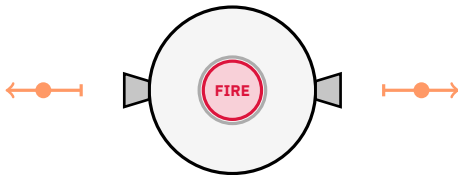
PARTICLE EMITTER

- ▶ This device is a *particle emitter*
- ▶ You can press the button labelled **FIRE**
- ▶ Two particles are emitted from it in opposite directions



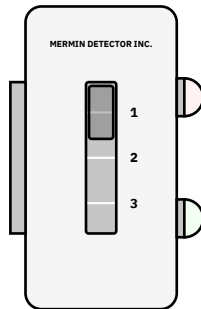
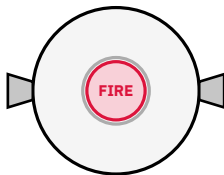
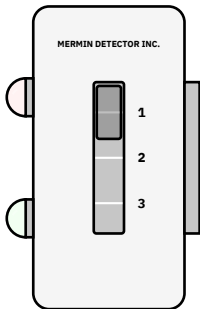
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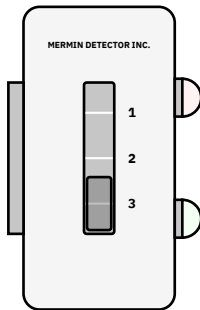
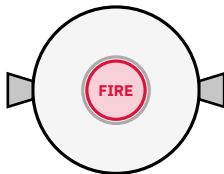
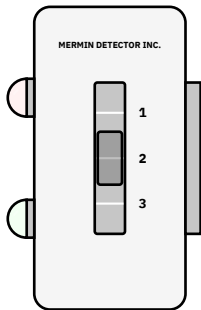
EXPERIMENTAL SETUP

1. Place the particle emitter between two Mermin detectors



EXPERIMENTAL SETUP

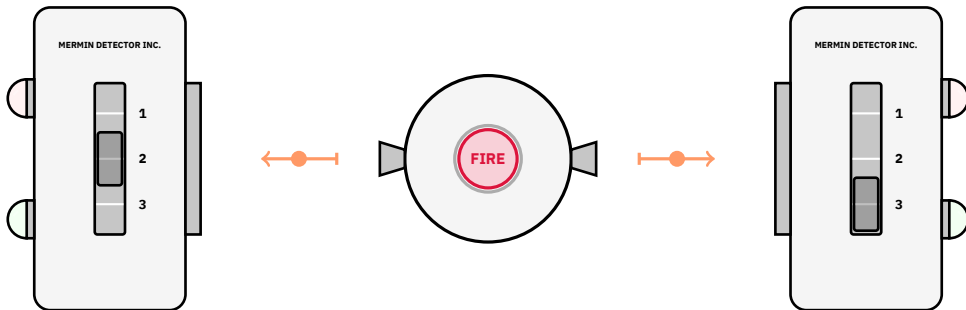
2. Randomly set the switches



There are 9 equally likely configurations: $1:1^L 1:2^R$, $1:1^L 1:3^R$, $2:1^L 2:2^R$, $2:1^L 2:3^R$, $3:1^L 3:2^R$, $3:1^L 3:3^R$ and $2:3^L$. Here we have $2:3^L$

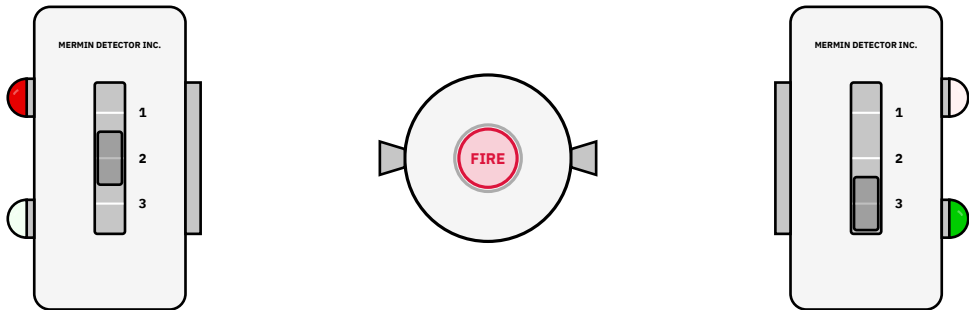
EXPERIMENTAL SETUP

3. Press the **FIRE** button — two particles are emitted



EXPERIMENTAL SETUP

4. The particles hit the detectors — observe which lights flash



Here we have **RED** and **GREEN**. We record this result as **23RG** — switches set at $2:3$ and flashes $R:G$

REPEAT, REPEAT, REPEAT

- ▶ Repeat these 4 steps over and over again — I will show results from 10,000 repeats
- ▶ We choose the switch settings randomly each time
- ▶ We record each result in the format **[LEFT SWITCH][RIGHT SWITCH][LEFT COLOR][RIGHT COLOR]**, e.g., **23RG** every time

Section 2

MERMIN EXPERIMENT RESULTS

NOTEBOOK OF FIRST 100 RESULTS

12RR	21RG	22RR	33RR	12GR	21GR	33RR	23RR	31RG	23RR
22RR	11RR	22GG	12GG	32GG	13RG	23GR	23RR	23RG	11RR
31GR	11GG	21GR	33GG	12RR	12RG	22GG	13RG	21GR	12RG
31RR	12GR	32GR	11RR	23RG	12GR	11RR	32RG	12RR	23RG
32GR	13GR	32RR	32GG	11GG	12GR	11GG	31RR	13GG	33RR
33RR	23RG	32RG	22GG	31RG	13RG	23RR	13GR	21RG	13RR
13RR	11RR	22RR	21RR	12RR	22GG	23RG	31RG	12GR	22RR
23RG	31GG	33GG	21RR	32RG	11GG	31RG	33GG	12RG	13GR
33RR	32GG	31RG	22RR	23RG	33GG	22GG	23GR	31GR	31GR
33RR	13GR	22RR	32RG	13RG	13GG	23RG	12RG	23RR	12GR

CASE (A)

Look at when the two detectors are in the same state, i.e. $\overset{L}{1}:\overset{R}{1}$, $\overset{L}{2}:\overset{R}{2}$ or $\overset{L}{3}:\overset{R}{3}$

12RR	21RG	22RR	33RR	12GR	21GR	33RR	23RR	31RG	23RR
22RR	11RR	22GG	12GG	32GG	13RG	23GR	23RR	23RG	11RR
31GR	11GG	21GR	33GG	12RR	12RG	22GG	13RG	21GR	12RG
31RR	12GR	32GR	11RR	23RG	12GR	11RR	32RG	12RR	23RG
32GR	13GR	32RR	32GG	11GG	12GR	11GG	31RR	13GG	33RR
33RR	23RG	32RG	22GG	31RG	13RG	23RR	13GR	21RG	13RR
13RR	11RR	22RR	21RR	12RR	22GG	23RG	31RG	12GR	22RR
23RG	31GG	33GG	21RR	32RG	11GG	31RG	33GG	12RG	13GR
33RR	32GG	31RG	22RR	23RG	33GG	22GG	23GR	31GR	31GR
33RR	13GR	22RR	32RG	13RG	13GG	23RG	12RG	23RR	12GR

CASE (A)

Case (A): The two detectors are in the same state, i.e. $\overset{L}{1}:\overset{R}{1}$, $\overset{L}{2}:\overset{R}{2}$ or $\overset{L}{3}:\overset{R}{3}$

1. This case occurs about one-third of the time as the detector states are randomly chosen — 3,301 times in 10,000 results
2. In this case, we only ever see $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$
3. We see them with approximately equal frequency — 1,636 occurrences of $\overset{L}{R}:\overset{R}{R}$ and 1,665 of $\overset{L}{G}:\overset{R}{G}$

Case (A): $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ occurs 100% of the time

CASE (B)

Look at when the two detectors are in different states, i.e. $\overset{L}{1}:\overset{R}{2}$, $\overset{L}{1}:\overset{R}{3}$ $\overset{L}{2}:\overset{R}{1}$ $\overset{L}{2}:\overset{R}{3}$ $\overset{L}{3}:\overset{R}{1}$ or $\overset{L}{3}:\overset{R}{2}$

12RR	21RG	22RR	33RR	12GR	21GR	33RR	23RR	31RG	23RR
22RR	11RR	22GG	12GG	32GG	13RG	23GR	23RR	23RG	11RR
31GR	11GG	21GR	33GG	12RR	12RG	22GG	13RG	21GR	12RG
31RR	12GR	32GR	11RR	23RG	12GR	11RR	32RG	12RR	23RG
32GR	13GR	32RR	32GG	11GG	12GR	11GG	31RR	13GG	33RR
33RR	23RG	32RG	22GG	31RG	13RG	23RR	13GR	21RG	13RR
13RR	11RR	22RR	21RR	12RR	22GG	23RG	31RG	12GR	22RR
23RG	31GG	33GG	21RR	32RG	11GG	31RG	33GG	12RG	13GR
33RR	32GG	31RG	22RR	23RG	33GG	22GG	23GR	31GR	31GR
33RR	13GR	22RR	32RG	13RG	13GG	23RG	12RG	23RR	12GR

CASE (B)

Case (B): the two detectors are in different states, i.e. $\overset{L}{1}:\overset{R}{2}$, $\overset{L}{1}:\overset{R}{3}$, $\overset{L}{2}:\overset{R}{1}$, $\overset{L}{2}:\overset{R}{3}$, $\overset{L}{3}:\overset{R}{1}$ or $\overset{L}{3}:\overset{R}{2}$

1. This case occurs about two-thirds of the time as the detector states are randomly chosen — 6,699 times in 10,000 results
2. In this case, we see all four outcomes: $\overset{L}{R}:\overset{R}{R}$, $\overset{L}{G}:\overset{R}{G}$, $\overset{L}{R}:\overset{R}{G}$ and $\overset{L}{G}:\overset{R}{R}$
3. In this case, we see matching colors, $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$, about 25% of the time — 1690 times out of 6,699 cases

Case (B): $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ occurs 25% of the time

SUMMARIZING OBSERVATIONS

We can summarize our results as follows:

- ▶ **Case (A):** When detector states are the *same*, we see $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ 100% of the time
- ▶ **Case (B):** When detector states are *different*, we see $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ 25% of the time

Section 3

HIDDEN INSTRUCTION MODEL

EXPLAINING OBSERVATIONS IN CASE (A)

Q: There are no wires or connections between the detectors. How can the detectors produce matching outcomes without communicating?

A: Hidden instruction model

PARTICLE CARRIES HIDDEN INSTRUCTIONS

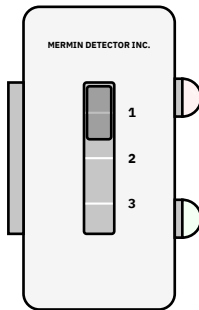
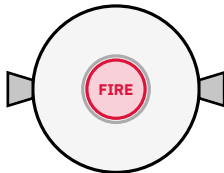
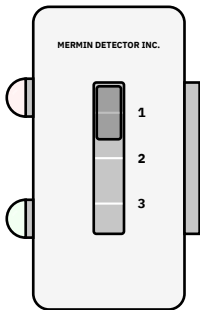


HIDDEN INSTRUCTION MODEL

- ▶ The particles contain hidden instructions, e.g., if state **1**, show **RED**
- ▶ The instructions could be encoded in the form e.g., $\overset{1}{R}\overset{2}{G}\overset{3}{R}$, which means
 - ▶ If state **1**, show **RED**
 - ▶ If state **2**, show **GREEN**
 - ▶ If state **3**, show **RED**
- ▶ There are 8 possible instruction sets: $\overset{1}{R}\overset{2}{R}\overset{3}{R}$, $\overset{1}{R}\overset{2}{R}\overset{3}{G}$, $\overset{1}{R}\overset{2}{G}\overset{3}{R}$, $\overset{1}{R}\overset{2}{G}\overset{3}{G}$, $\overset{1}{G}\overset{2}{R}\overset{3}{R}$, $\overset{1}{G}\overset{2}{R}\overset{3}{G}$, $\overset{1}{G}\overset{2}{G}\overset{3}{R}$ and $\overset{1}{G}\overset{2}{G}\overset{3}{G}$
- ▶ The coordination is explained if the emitter sends the *same* instruction to each detector

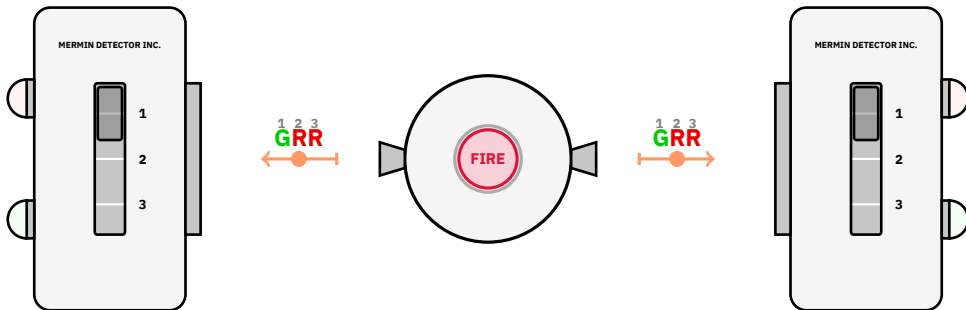
EMITTER SENDS INSTRUCTIONS

1. The detectors are in the same state — $\overset{L}{1}:\overset{R}{1}$



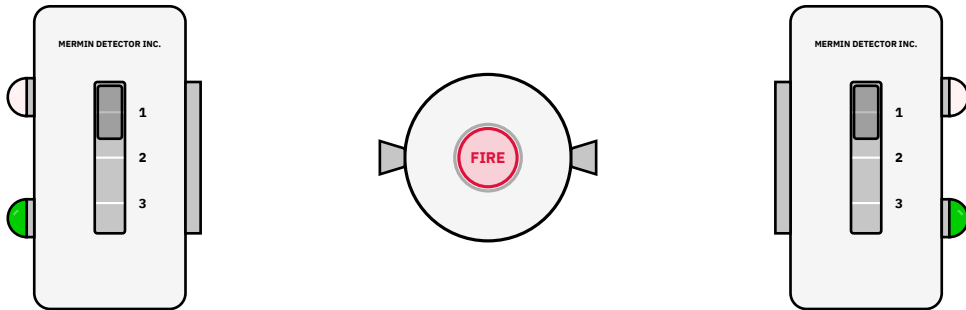
EMITTER SENDS INSTRUCTIONS

2. We press **FIRE** and two particles are emitted carrying *identical instructions*, e.g., ^{1 2 3}GRR



EMITTER SENDS INSTRUCTIONS

3. The detectors follow the instructions and thus show the same colour



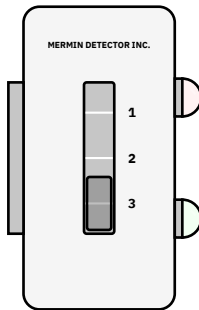
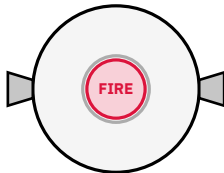
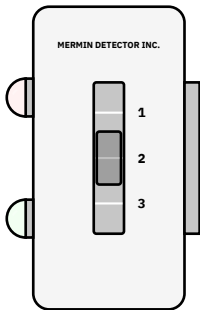
The detectors were in state **1** and received instruction ^{1 2 3}**GRR** and thus showed **GREEN**

HIDDEN INSTRUCTION MODEL IN CASE (B)

- ▶ The emitter doesn't know the detector settings — the hidden instructions it sends cannot depend on the detector settings
- ▶ Suppose the emitter again sends $\overset{1}{\text{G}}\overset{2}{\text{R}}\overset{3}{\text{R}}$ but this time the switches are in different states, e.g., $\overset{\text{L}}{2}:\overset{\text{R}}{3}$

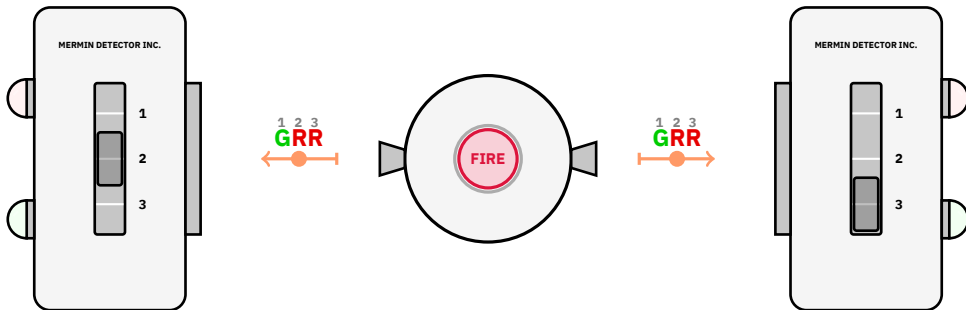
EMITTER SENDS INSTRUCTIONS

1. The detectors are in different states — $2^L 3^R$



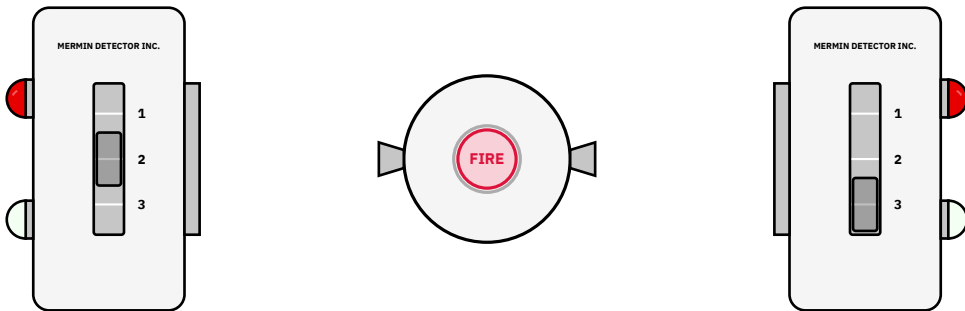
EMITTER SENDS INSTRUCTIONS

2. We press **FIRE** and two particles are emitted carrying identical instructions, e.g., ^{1 2 3}**GRR**



EMITTER SENDS INSTRUCTIONS

3. The detectors follow the instructions



The detectors were in state $\overset{L}{2}:\overset{R}{3}$ and received instruction $\overset{1}{G}\overset{2}{R}\overset{3}{R}$. They thus showed the colors corresponding to the second and third characters, both **RED**

HIDDEN INSTRUCTION MODEL IN CASE (B)

For the code $\overset{1}{\text{G}}\overset{2}{\text{R}}\overset{3}{\text{R}}$ and different detector states, how often will they flash the same colour?

- ▶ There are 6 equally probable switch settings: $\overset{\text{L}}{1}:\overset{\text{R}}{2}$, $\overset{\text{L}}{1}:\overset{\text{R}}{3}$, $\overset{\text{L}}{2}:\overset{\text{R}}{1}$, $\overset{\text{L}}{2}:\overset{\text{R}}{3}$, $\overset{\text{L}}{3}:\overset{\text{R}}{1}$ and $\overset{\text{L}}{3}:\overset{\text{R}}{2}$
- ▶ $\overset{\text{L}}{\text{G}}:\overset{\text{R}}{\text{G}}$ is impossible, as it would require setting $\overset{\text{L}}{1}:\overset{\text{R}}{1}$
- ▶ $\overset{\text{L}}{\text{R}}:\overset{\text{R}}{\text{R}}$ is possible for 2 switch settings: $\overset{\text{L}}{2}:\overset{\text{R}}{3}$ and $\overset{\text{L}}{3}:\overset{\text{R}}{2}$
- ▶ Thus they show the same colour with a frequency of $2/6$, about 33%

The same reasoning applies to the 5 codes $\overset{1}{\text{R}}\overset{2}{\text{G}}\overset{3}{\text{R}}$, $\overset{1}{\text{R}}\overset{2}{\text{R}}\overset{3}{\text{G}}$, $\overset{1}{\text{G}}\overset{2}{\text{G}}\overset{3}{\text{R}}$, $\overset{1}{\text{G}}\overset{2}{\text{R}}\overset{3}{\text{G}}$ and $\overset{1}{\text{R}}\overset{2}{\text{G}}\overset{3}{\text{G}}$

EXPLAINING OBSERVATIONS IN CASE (B)

For the code $\overset{1}{\text{R}}\overset{2}{\text{R}}\overset{3}{\text{R}}$, how often will they flash the same colour?

► $\overset{\text{L}}{\text{R}}:\overset{\text{R}}{\text{R}}$ occurs for any switch setting, i.e., at a frequency of 100%

The same reasoning applies to the code $\overset{1}{\text{G}}\overset{2}{\text{G}}\overset{3}{\text{G}}$. We have now considered all 8 codes

CONTRADICTION IN THE HIDDEN INSTRUCTION MODEL

- ▶ Our explanation for observations in **Case (A)** was that the particles carry the same instruction set to each detector — the hidden instruction model
- ▶ We have just shown that this predicts $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ should occur at least 33% of the time in **Case (B)**
- ▶ Observations showed that $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ occurred only 25% of the time in **Case (B)**

Conclusion: Identical hidden instructions cannot explain the data

Section 4

BELL'S THEOREM

WHAT CAN EXPLAIN THE DATA?

There were 3 assumptions in our failed hidden instruction model

- ▶ **Locality:** the output from the left-hand side detector cannot depend on the right-hand side detector setting, and vice-versa
- ▶ **Realism:** the detectors are reacting to pre-existing instructions carried by the particles
- ▶ **No-conspiracy:** we were free to choose the detector settings — no conspiracy between our setting choices and the particles emitted

We could break one of these to explain the data

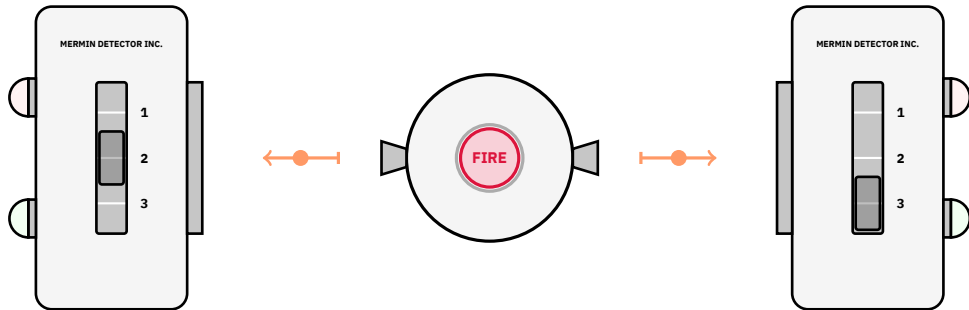
LOCALITY

Locality: the output from the left-hand side detector cannot depend on the right-hand side detector setting, and vice-versa

- ▶ In the hidden instruction model, the detector output depended *only* on the detector setting and the instruction
- ▶ E.g., detector setting **1** and instruction ^{1 2 3}**RGG** always outputs **RED** regardless of the other detector setting
- ▶ Non-local instructions would depend on the states of *both* detectors. In a non-local hidden instruction model, each particle could carry an instruction for the colours of each detector in the 9 possible cases, e.g., ^{1:1}**RR** ^{1:2}**GR** ^{1:3}**GG** ^{2:1}**RG** ^{2:2}**RR** ^{2:3}**GR** ^{3:1}**RR** ^{3:2}**GR** ^{3:3}**GG**

EXERCISE: CONSTRUCT INSTRUCTION SETS FOR A NON-LOCAL HIDDEN INSTRUCTION MODEL

LOCALITY



What happens in this detector ...

**... cannot depend on this detector setting,
arbitrarily far away**

REALISM

Realism: the detectors are reacting to pre-existing instructions carried by the particles

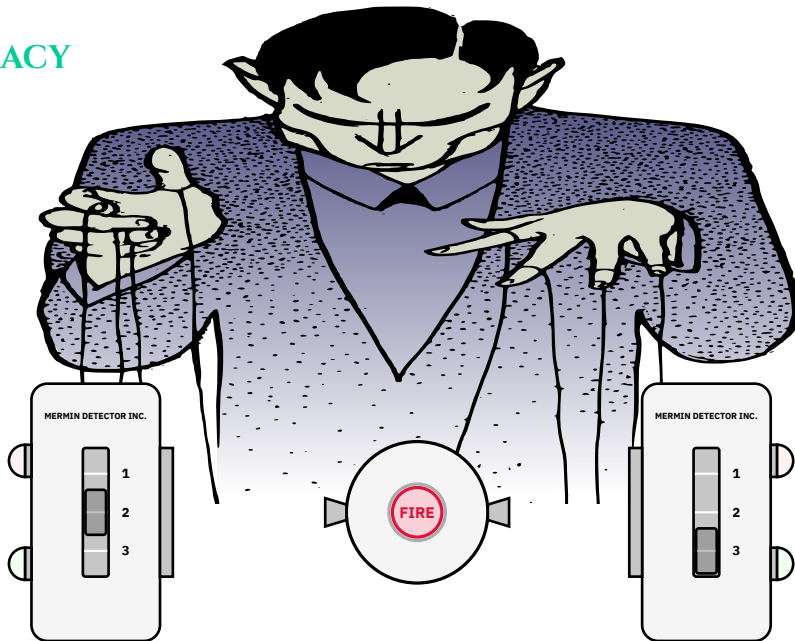
- ▶ In the hidden instruction model, the particles carried a pre-existing instruction, e.g., ¹**R**²**G**³**G**
- ▶ With detector setting **1**, we see **RED**. The instructions ²**G**³**G** for cases **2** and **3** existed, even though we didn't choose settings **2** or **3**

NO-CONSPIRACY

No-conspiracy: we were free to choose the detector settings — no conspiracy between our setting choices and the particles emitted

- ▶ For any hidden instruction emitted, e.g., $\overset{1}{R}\overset{2}{G}\overset{3}{G}$, we were free in principle to have chosen any of the 9 detector settings: $\overset{L}{1}:\overset{R}{1}$, $\overset{L}{1}:\overset{R}{2}$, $\overset{L}{1}:\overset{R}{3}$, $\overset{L}{2}:\overset{R}{1}$, $\overset{L}{2}:\overset{R}{2}$, $\overset{L}{2}:\overset{R}{3}$, $\overset{L}{3}:\overset{R}{1}$, $\overset{L}{3}:\overset{R}{2}$ or $\overset{L}{3}:\overset{R}{3}$
- ▶ A conspiracy would mean we were not free to choose the detector settings — we were fated to choose the ones we did given the instructions of the particles that were emitted

CONSPIRACY



CONSPIRACY

- ▶ Suppose that the puppetmaster picked a random row from this table
- ▶ The result was fated to be the one in that row
- ▶ We weren't free to choose the settings — we were fated to choose the settings in that row
- ▶ We would see $\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$ in 100% in **case (A)** and about 17% in **case (B)**
- ▶ That is less than the 33% bound we found before — with a bigger table, we could even arrange exactly 25%

Setting	Case	Result	$\overset{L}{R}:\overset{R}{R}$ or $\overset{L}{G}:\overset{R}{G}$
$\overset{L}{1}:\overset{R}{1}$	A	$\overset{L}{R}:\overset{R}{R}$	Y
$\overset{L}{1}:\overset{R}{2}$	B	$\overset{L}{R}:\overset{R}{R}$	Y
$\overset{L}{1}:\overset{R}{3}$	B	$\overset{L}{R}:\overset{R}{G}$	N
$\overset{L}{2}:\overset{R}{1}$	B	$\overset{L}{G}:\overset{R}{R}$	N
$\overset{L}{2}:\overset{R}{2}$	A	$\overset{L}{G}:\overset{R}{G}$	Y
$\overset{L}{2}:\overset{R}{3}$	B	$\overset{L}{R}:\overset{R}{G}$	N
$\overset{L}{3}:\overset{R}{1}$	B	$\overset{L}{R}:\overset{R}{G}$	N
$\overset{L}{3}:\overset{R}{2}$	B	$\overset{L}{G}:\overset{R}{R}$	N
$\overset{L}{3}:\overset{R}{3}$	A	$\overset{L}{R}:\overset{R}{R}$	Y

**EXERCISE: CONSTRUCT A TABLE OF 18 ENTRIES THAT YIELDS A
25% PROBABILITY**

BELL'S THEOREM

We have reached *Mermin's version* of *Bell's theorem* (Bell, 1964)

Bell's theorem: Without a *conspiracy*, no theory that is both *local* and *real* can explain this data

This is profoundly important: the data rule out any hidden instruction model and any local-real theory. Whatever is left will be weird and non-classical

WHICH ASSUMPTION SHOULD WE BREAK?

To explain the data, which assumption should we break?

- ▶ **Locality:** breaking it is in tension with *special relativity* and allows dependence at arbitrary distances
- ▶ **Realism:** breaking it means that properties of objects do not exist before they are measured
- ▶ **No-conspiracy:** breaking it means that our choices of measurement settings can never be truly free

Section 5

WELCOME TO THE QUANTUM WORLD

WELCOME TO THE QUANTUM WORLD

- ▶ **Quantum mechanics** can explain this data extremely successfully

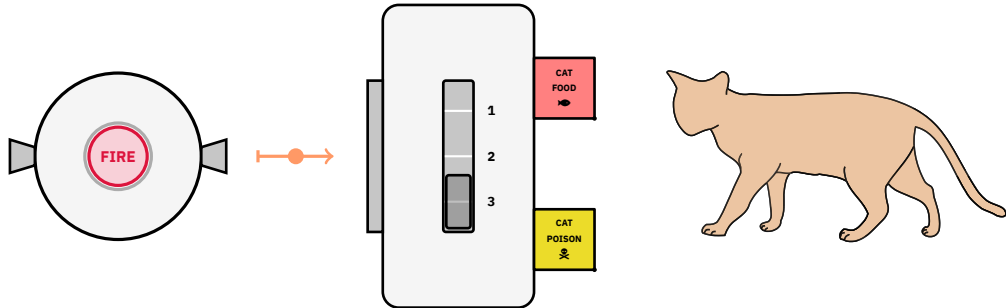
Non-real: Quantum mechanics breaks realism

- ▶ That means that some properties do not have definite values before we measure them
- ▶ Schrödinger's cat is a thought experiment that shows us just how weird that would be if it applied to everyday objects

This is a thought experiment — I oppose any cruelty to animals

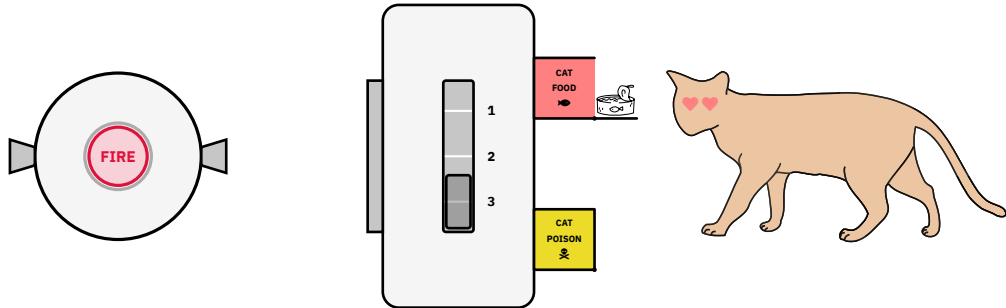
SCHRÖDINGER'S CAT

Instead of red and green lights that can flash, this device has cat food and cat poison boxes that can open and a cat



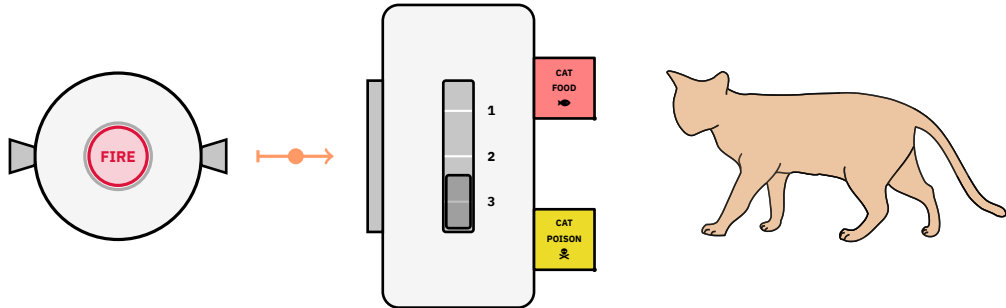
SCHRÖDINGER'S CAT

Instead of red and green lights that can flash, this device has **cat food** and cat poison boxes that can open and a cat



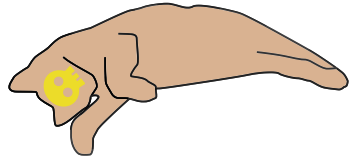
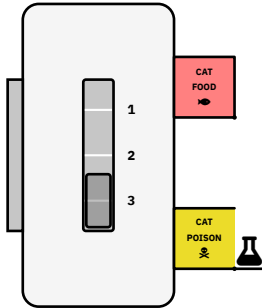
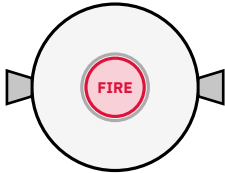
SCHRÖDINGER'S CAT

Instead of red and green lights that can flash, this device has cat food and cat poison boxes that can open and a cat



SCHRÖDINGER'S CAT

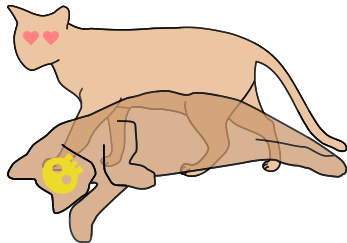
Instead of red and green lights that can flash, this device has cat food and **cat poison** boxes that can open and a cat



SCHRÖDINGER'S CAT

- ▶ In quantum theory, some properties do not exist until they are observed
- ▶ If we press **FIRE**, but don't observe the cat or the detector, is the cat dead or alive?
- ▶ If quantum rules applied to everyday objects as well as small things, before the cat is observed, it must be in a strange state — **a superposition** — of dead **and** alive

Superposition of dead and alive
is weird — resolving that
weirdness is known as the
measurement problem



FURTHER READING

- ▶ Mermin's original paper (Mermin, 1981) and follow-up Mermin (1990)
- ▶ This simple proof of Bell's inequality (Maccone, 2013)
- ▶ The famous Einstein-Podolsky-Rosen (EPR) paper (Einstein et al., 1935)
- ▶ John Bell's book (Bell, 2004)

CONCLUSIONS

- ▶ We described a simple experiment involving a particle emitter and two detectors
- ▶ We showed that the results could not be explained by a simple local realist theory
- ▶ To explain the results, we needed to give up at least one assumption
 - ▶ **Locality** — measurements cannot depend on things that are arbitrarily far away
 - ▶ **Realism** — measurements reveal pre-existing properties of objects
 - ▶ **No-conspiracy** — we are free to choose what measurements we make
- ▶ Quantum mechanics gives up realism — some properties do not have pre-existing values before we observe them
- ▶ Schrödinger's cat shows us just how weird quantum mechanics would be if it applied to everyday objects



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