Relative plausibility of scientific theories: WIMP dark matter

Figures and results from C. Balazs, J. Ellis, A. Fowlie, L. Marzola, and M. Raidal, (2017), arXiv:1711.09912 [hep-ph]

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30 November 2017. A Fractured Universe? Fundamental Physics, Symmetry and Life

Monash University

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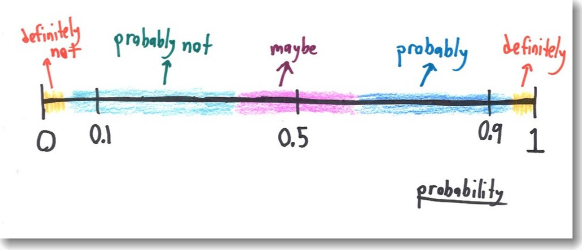
Nutshell

What is probability?

A measure of plausibility.



Probability $\Leftrightarrow$ plausibility

Games of chance

We are familiar with the application of probability to games of chance.

What is the probability of heads?

Games of chance

We are familiar with the application of probability to games of chance.

What was the probability of being dealt this winning hand?

Games of skill

We are suspicious of the application of probability to games of skill.

What was the probability that the machine would beat the human?

Games of skill

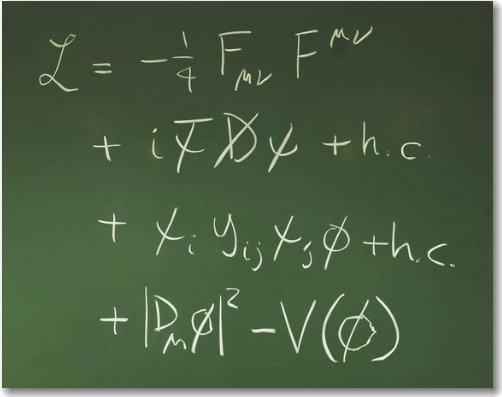
We are suspicious of the application of probability to games of skill.

Though perhaps accept it when gambling is common.

What was the probability that Leicester would win the Premier League?

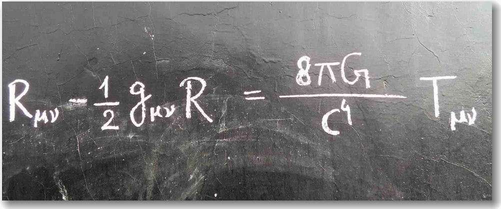
What about applying it to scientific theories?

What is the probability of this theory in light of LHC experiments?



What about applying it to scientific theories?

What about this one in light of LIGO's discoveries?



Prior knowledge

All of them depend upon priors

The probability of a heads from the toss of a coin depends on prior belief about the dynamics and initial conditions of the coin.

Prior knowledge

All of them depend upon priors

The probability of a winning hand depends on prior belief about the shuffled pack of cards and the integrity of the dealer.

Prior knowledge

All of them depend upon priors

The probability of victory in go or a football match depends on prior beliefs about the skills of the players and models for the outcome.

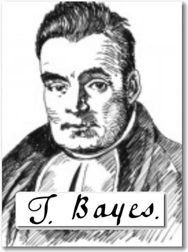
All of them depend upon priors

The probability of a scientific theory in light of data depends on prior beliefs about the theory's parameters, the theory itself and alternative theories.

Bayesian statistics

Bayesian statistics

Bayesian statistics is a mathematical framework for describing plausibility — a calculus of beliefs [2].



Developed by Bayes, Laplace and Jeffreys in 18th, 19th and 20th centuries.

Bayes' theorem

The most important equation is Bayes' theorem — a unique rule for updating plausibility in light of data:

$$p(M \mid D) = \frac{p(D \mid M)}{p(D)} \cdot p(M).$$

Our posterior belief in a model, M, is found by updating our prior belief with data, D.

Model comparison

To update our belief in a model in light of data, we must consider more than one model.

If we believe absolutely in a single model, we obtain

$$p(M \mid D) = p(M) = 1.$$

We simply find that we are certain about the model before and after data.

Bayes factors

Thus we must compare models. We compare two models with a so-called Bayes factor

$$\text{Bayes factor} = \frac{\text{Relative plausibility after data}}{\text{Relative plausibility before data}}$$

in maths, by Bayes' theorem,

$$\text{Bayes factor} = \underbrace{\frac{p(D \mid M_a)}{p(D \mid M_b)}}_{\text{Calculate this ratio}} = \frac{\overbrace{\frac{p(M_a \mid D)}{p(M_b \mid D)}}^{\text{Posterior odds — output}}}{\underbrace{\frac{p(M_a)}{p(M_b)}}_{\text{Prior odds — input}}}$$

Bayesian evidence

A Bayes factor is itself a ratio of evidences, where

$$\text{Evidence} = p(D \mid M) = \int p(D \mid M, x) \cdot p(x \mid M) \, dx$$

The integrand is a product of likelihood and prior. Likelihood could be e.g. a Gaussian for Higgs mass measurement or Planck measurement of the dark matter relic density.

The integration is over the model's parameters x. The integration may be computationally challenging.

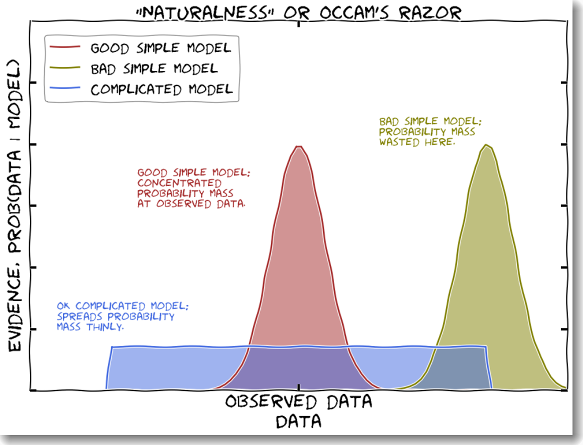
Fine-tuning

We always evaluate evidences with the observed data; nevertheless, the probability of any data must equal one,

$$p(\text{any data} \mid M) = \int p(D \mid M)\, dD = 1.$$

Thus models have a finite probability mass to spend on their predictions.

This means that fine-tuned models and models that make diffuse predictions are penalised.



Fine-tuning

This is an automatic Occam's razor/penalty for fine-tuning. Complicated theories make diffuse predictions. Fine-tuned theories make generically bad predictions. They are relatively implausible.

See Csaba's talk tomorrow or my talk last year.

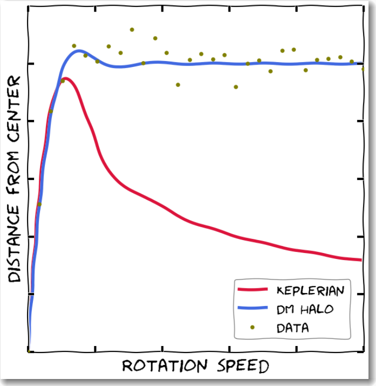
Example

Since virtue cannot be taught, only demonstrated, I now present an example with models of dark matter.

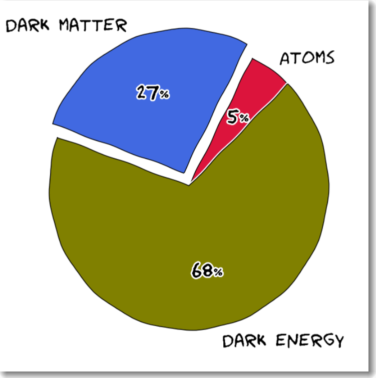
Dark matter experimental evidence and constraints

Dark matter experimental evidence

We all know the evidence for dark matter (DM) in gravitational interactions, e.g.



(I) Rotation curves [3]



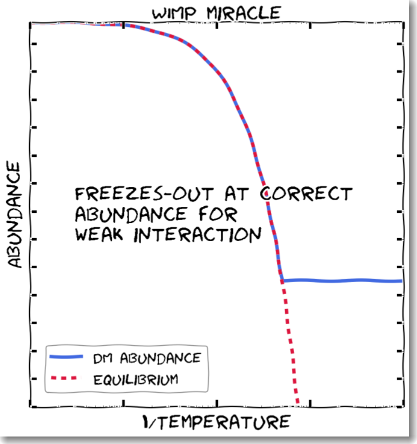
(II) CMB [4]

Once it is cold enough, DM particles cannot overcome Hubble expansion and thus cannot annihilate.

This freeze-out of thermal equilibrium with bath of Standard Model (SM) particles sets relic density.

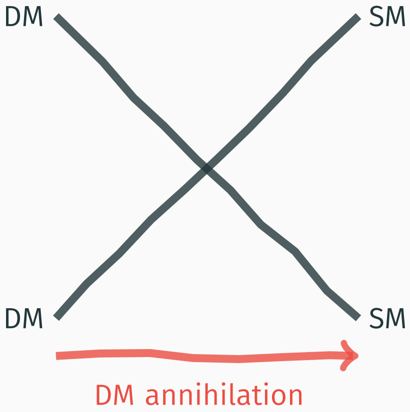
WIMP miracle

This is the WIMP miracle — as correct density achieved for weak interactions.



DM annihilates to SM

DM must annihilate in the early Universe to set the relic density measured by Planck.



Relic abundance

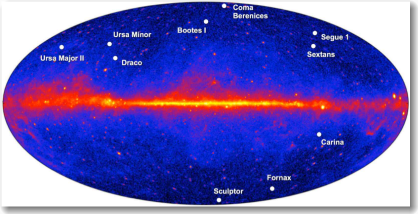
From measurements of the CMB Planck [4] found

$$\text{Relic abundance} = \Omega h^2 = 0.1199 \pm 0.0022$$

in ΛCDM. We use a Gaussian likelihood.

Indirect detection

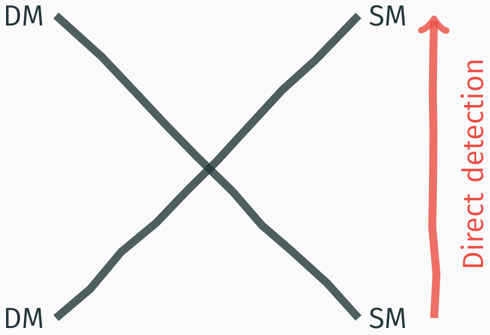
DM annihilation could result in signals from high mass-to-light galaxies such as dwarf spheroidal galaxies.



Fermi-LAT [5] searched for a γ-ray signal but saw nothing, resulting in constraints on DM annihilation cross section.

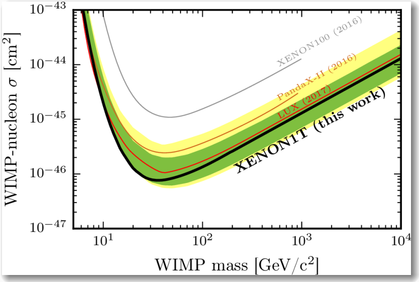
DM scatters with SM

We can search for DM in direct detection experiments. DM elastic scatters with nucleons in a detector on Earth.



There is a wind of WIMP particles from the Earth's motion in the dark matter halo.

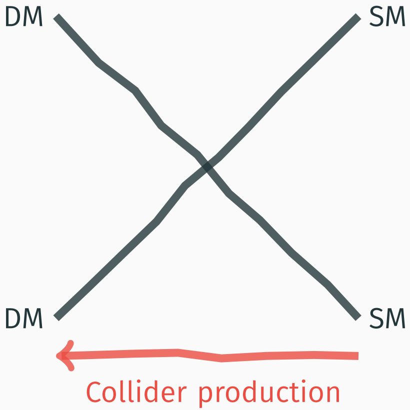
The Panda [6], LUX [7], XENON [8] and PICO [9] experiments saw nothing, resulting in exclusion contours on the (mass, cross section) planes:



Our likelihood function for this data was a step-function. We included uncertainty in nuclear form factors and the local density.

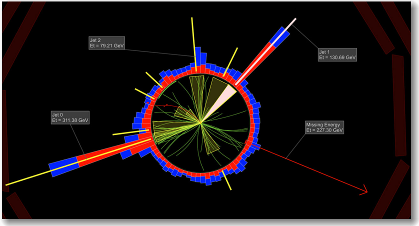
SM annihilates to DM

We can search for DM produced from collisions of SM particles.



Collider searchers

The LHC [10] saw nothing — wanted to find missing energy as DM escapes from the detector.



LEP [11] saw nothing — wanted to find Z decaying into DM particles.

Data

Ωh^2	$0.1199 \pm 0.0022 \pm 10\%$	Planck [4]
Γ_Z^{inv}	$499.0 \pm 1.5 \pm 0.014\,\mathrm{MeV}$	LEP [12]
$\mathrm{BR}_h^{\mathrm{inv}}$	$\lesssim 0.24$	LHC [13]
$\sigma_{\mathrm{SI}}^{p,n}$	$\lesssim 10^{-46}\mathrm{cm}^2$	PandaX [6]
σ_{SD}^{n}	$\lesssim 10^{-40}\mathrm{cm}^2$	PandaX [14]
σ_{SD}^{p}	$\lesssim 10^{-40}\mathrm{cm}^2$	PICO [7]
$\langle \sigma v \rangle$	$\lesssim 10^{-26}\mathrm{cm}^3/\mathrm{s}$	IceCube [5]
Mono-X searches	$\sqrt{s} = 8\,\mathrm{TeV}$ and $13\,\mathrm{TeV}$	LHC [10]

Waning of the WIMP?

In light of the failure to discover DM in direct detection experiments, many doubting the plausibility of WIMP DM.

WIMP DM models can be fine-tuned to agree with data but was their plausibility damaged?

Simple theories of dark matter

Simplest theories of DM

The simplest WIMP models of DM add a single particle to the SM: the WIMP itself.

The WIMP interacts with SM by a Z or Higgs portal:

We consider all renormalizable, Lorentz invariant interactions for WIMPs with spin-0, 1/2 and 1.

SM portal models

There are many models

(scalar, Majorana fermion, Dirac fermion, vector) spin of WIMP
$\times$ (Higgs, Z) mediator

We added them all to the DM program `microMEGAs` [15, 16] via the model building program `calcHEP` [17].

Priors for DM mass and couplings

We picked logarithmic priors for DM mass and coupling, since we are ignorant of their scale.

DM mass, m_χ	1 GeV – 10 TeV	Log
DM coupling with SM, g	10^{-4} – 4π	Log

There is a sensitivity analysis with linear priors in the paper.

Priors nuisance parameters

DM scattering rate with matter depends upon nuclear form factors.

Nuclear			
σ_s	$41.1 \pm 8.1^{+7.8}_{-5.8}$ MeV	Lattice, ETM [18]	Gaussian
$\sigma_{\pi N}$	$37.2 \pm 2.6^{+4.7}_{-2.9}$ MeV	Lattice, ETM [18]	Flat + tails
	58 ± 5 MeV	Pheno [19]	
m_u/m_d	0.38 – 0.58	Lattice [12]	Flat
m_s/m_d	17 – 22	Lattice [12]	Flat

We also investigated an alternative treatment of $\sigma_{\pi N}$.

Priors nuisance parameters

DM flux on Earth depends on density and velocity distribution of DM.

Astrophysical		
ρ_{DM}	$0.3\,\mathrm{GeV/cm^3}$	Log-normal
v_{esc}	$550 \pm 35\,\mathrm{km/s}$	Gaussian
v_{rel}	$235 \pm 20\,\mathrm{km/s}$	Gaussian
v_0	$235 \pm 20\,\mathrm{km/s}$	Gaussian
J-factor for dSphs		Log-normal [5]

Priors nuisance parameters

DM annihilation sensitive to masses of Higgs and Z-boson.

SM			
M_Z	$91.1876 \pm 0.0021\,\text{GeV}$	Gaussian	LHC [12]
m_h	$125.09 \pm 0.24\,\text{GeV}$	Gaussian	LEP [12]

Relative plausibility of simple theories of dark matter

Relative plausibility of DM models

We now have

- Models, M_i: Scalar, fermion or vector DM that interacts with SM by Z or Higgs boson
- Data, D: Planck measurement of the relic density and failed searches for DM in direct detection, indirect detection and colliders
- Framework for relative plausibility: with Bayesian statistics we can calculate $p(M_i \mid D) / p(M_j \mid D)$

We calculated the evidence integrals with `MultiNest` [20–22].

Relative plausibility of DM models

We want to find change in relative plausibility of simple DM models in light of data.

Two models are implausible but there is no clear favourite between the rest.

Relative plausibility of DM models

Model	Bayes factor	$\min \chi^2$	p-value
Real scalar h-portal	0.55	2.6	0.27
Complex scalar h-portal	0.28	2.6	0.27
Real vector h-portal	0.23	2.6	0.27
Complex vector h-portal	0.059	2.6	0.27
Majorana h-portal	0.59	2.6	0.27
Dirac h-portal	0.71	2.6	0.27
Scalar Z-portal	3×10^{-14}	55	1.4×10^{-12}
Vector Z-portal	6.8×10^{-10}	35	2.2×10^{-8}
Majorana Z-portal	1	2.6	0.27
Dirac Z-portal	0.24	2.6	0.27

Relative plausibility of DM models

The vector Z and scalar Z portal models predicted substantial scattering cross sections.

Damage to simple DM models

Perhaps the failed searches for DM in direct detection experiments damage plausibility of our simple WIMP models?

Let's compare against an hypothetical model that predicts no signature in DD experiments.

Damage to simple DM models

Model	Damage to plausibility from DD		
	Present	Future	Neutrino floor
Real scalar h-portal	0.3	0.006	5×10^{-5}
Complex scalar h-portal	0.1	0.002	1×10^{-5}
Real vector h-portal	0.1	0.0009	9×10^{-7}
Complex vector h-portal	0.02	0.001	6×10^{-10}
Majorana h-portal	0.2	0.2	0.1
Dirac h-portal	0.2	0.1	0.1
Scalar Z-portal	1×10^{-14}	7×10^{-73}	7×10^{-129}
Vector Z-portal	3×10^{-10}	7×10^{-54}	2×10^{-101}
Majorana Z-portal	0.3	0.2	0.1
Dirac Z-portal	0.08	0.04	0.01

Damage to simple DM models

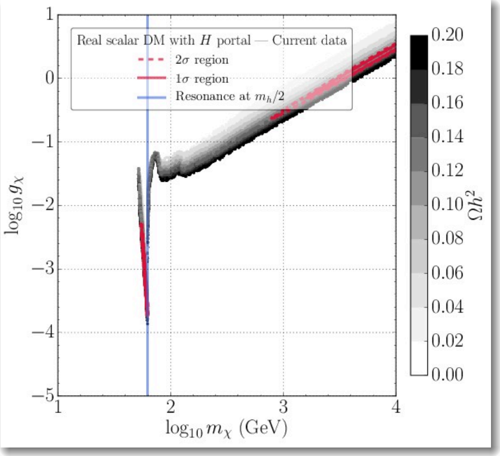
Direct detection experiments did not greatly damage the plausibility of many of the simplest models!

Hypothetical future results from LZ, XENONnT, and PICO might begin to damage a few models.

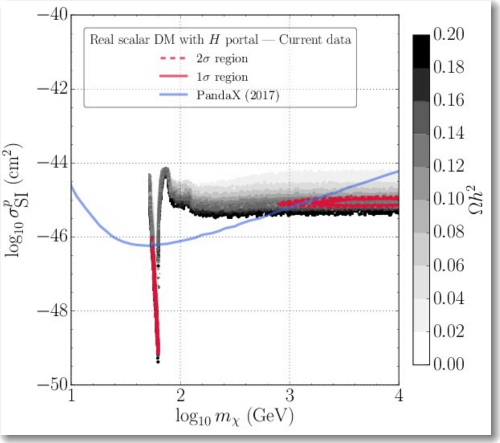
But fermionic models survive even once limits on the spin-independent cross section reach the neutrino floor!

Posteriors for the mass and couplings

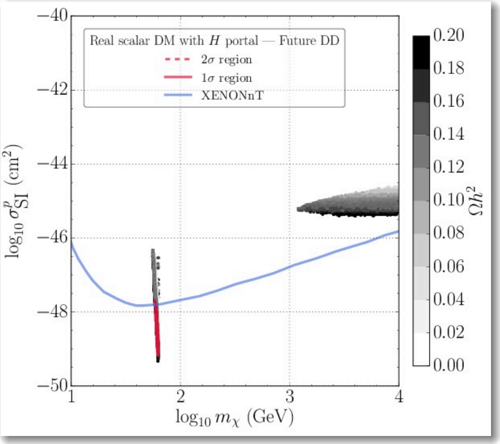
The mass of scalar DM with a Higgs portal is pushed to multi-TeV region in red or the narrow resonance region.



We require sensitivity for multi-TeV dark matter and/or low cross sections — future experiment XENONnT [23] should probe it.



DM is pushed into the Higgs funnel by XENONnT (but by this point this model becomes relatively implausible).



Conclusions

- We can compare plausibility of scientific theories with Bayesian statistics
- Automatic Occam's razor/penalty for fine-tuning
- I presented simple DM models as an example (see Ref. [1])
- Calculated changes in relative plausibility SM portal models in light of data
- Some the simplest WIMP models weren't badly damaged by failed searches for DM
- Waning of the WIMP is premature

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