Naturalness of the relaxion mechanism

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Very new work!
Just appeared at
arXiv:1602.03889
Fowlie, Balazs &
White (Monash)
Raidal & Marzola
(Tallinn)

Naturalness of the relaxion mechanism

1. Background: hierarchy problem etc
2. Relaxion model (appears to solve it)
3. Methodology of evaluating plausibility of models
4. Findings: compare relaxion vs. SM

relaxation with an axion

The relaxion: a clever new idea...

arXiv:1504.07551
Kaplan et al

- "It's a very, very clever idea" - Raman Sundrum
- "It's definitely clever" - Nima Arkani-Hamed

See also Abbot (1985), Dvali et al (2004, 2006) for related ideas

Hierarchy problem

No time for discussions about dim reg, whether cut-off physical, SM fundamental, classical scale invariance, blah blah blah

- Known since 1970s (Wilson (unpublished), Gildener (1976), Susskind etc)
- No symmetries to protect scalar-field mass in SM from quantum corrections
- High-scale/Planck-scale loop corrections
- SM generic prediction **WEAK SCALE ~ PLANCK SCALE**
- Our observation **WEAK SCALE <<< PLANCK SCALE**
- Of course, we could fine-tune s.t. everything agrees with observation

Relaxion model

arXiv:1504.07551
Phys. Rev. Lett. 115, 221801 (2015)
Kaplan et al

- Special interplay between (axion-like) scalar-field called relaxion and SM Higgs

Periodic barrier

SM quartic

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right) h^2 - m_b^3\langle h\rangle \cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4,$$

Field-dependent mass

Linear slope

Backreaction to VEV!

Realistic?

Further model building and discussion, see eg..

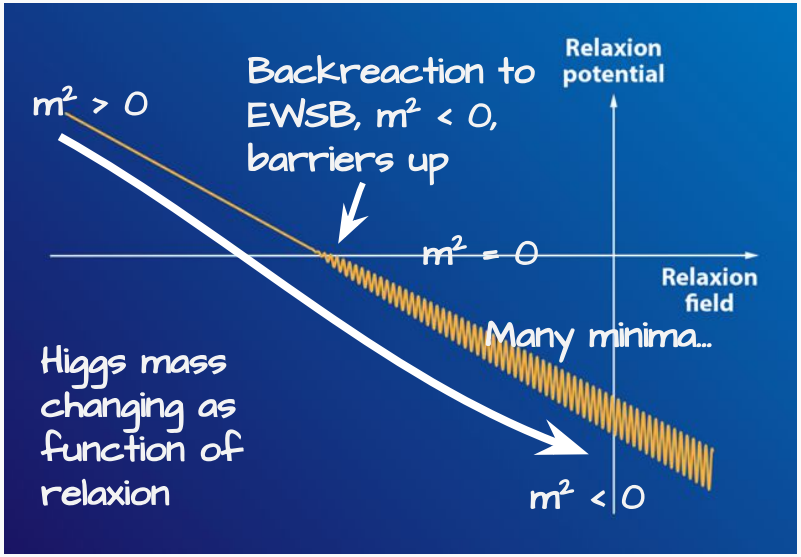
Giudice et al 1601.07183
Fonseca et al 1601.07183
Gupta et al 1512.00025
Ibanez et al 1509.00047
Matsedonskyi 1509.03583
Kaplan & Rattazzi 1511.01827
Choi & Im 1511.0013
Antipin & Redi 1509.00834

- Periodic term is axion-like - in QCD, generated from instanton dynamics
- Written coefficient as VEV of spurion that breaks shift symmetry (this could be very small)
- All other masses are unprotected and should all be close to the Planck scale
- Doubts about UV completion of this model (cannot be string inspired, underlying broken gauge symmetry, large field excursion)
- Agnostic/pragmatic: investigate minimal model and see if it works

Relaxion ingredients

arXiv:1504.07551
Kaplan et al
Figure: APS/Alan Stonebraker and Kaplan et al.

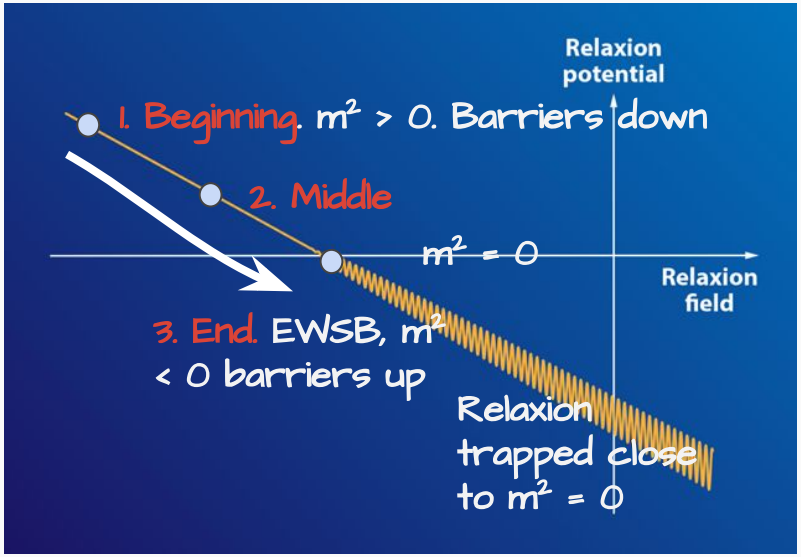
- Periodic component of potential: many minima, including minima with weak scale << Planck scale
- Backreaction: backreaction to EWSB affects shape of potential
- Dynamic Higgs mass: Higgs mass is a function of relaxion field
- Dissipates energy due to Hubble friction



Relaxion story

arXiv:1504.07551
Kaplan et al
Figure: APS/Alan Stonebraker and Kaplan et al.

- Beginning: relaxion large field value. EW unbroken
- Middle: relaxion rolls down linear potential, eventually triggering EWSB
- End: backreaction raises periodic barriers, trapping relaxion in a minima “close” to EWSB, i.e. at weak scale <<< Planck scale



No more pictures, now calculating stuff...

- So far calculations in literature hand-wavy (which is fine for describing general idea)
- Expressions for weak scale and theta QCD involve squiggles
- We minimize potential for Higgs and relaxion fields to find transcendental equation (where h is a function of phi):

$$\sin(\phi/f) = \frac{f\kappa\langle a\rangle}{m_b^3}\left(\frac{m^2/\kappa + \langle h\rangle^2}{\langle h\rangle}\right).$$

- This is OK (admittedly it would be better if we evolved the fields from some initial conditions with the EOM)

Solving transcendental by graphing

- Solution LHS = RHS at red star
- Simple analytic expression (brown hexagon - green diamond)
- Narrow enough for my calculations
- Assume relaxion stops in first minimum

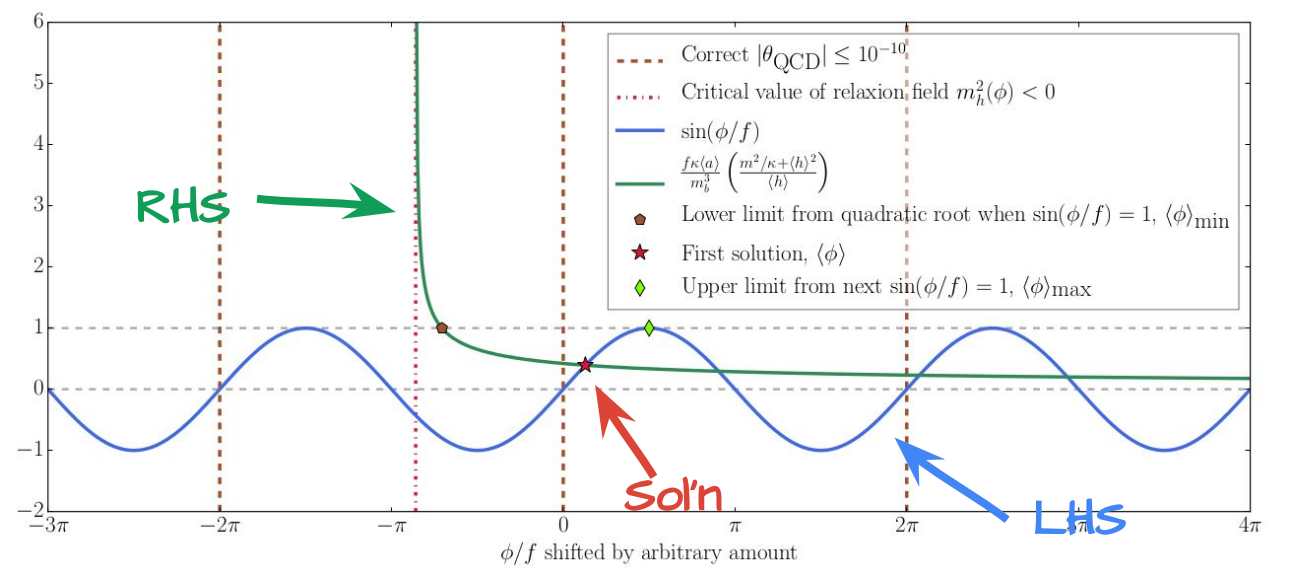


Fig. 1. in arXiv:1602.03889

Solving transcendental by graphing

- Find analytic approximations for weak scale. Confirm literature approximations in phenomenologically viable regimes:

$$\langle h \rangle_{\min} \approx f \frac{m^2 \langle a \rangle}{m_b^3}$$

- Solve for theta QCD in a similar manner: $\pi/2 - \sqrt{2\epsilon} + \cdots$
- Literature (seems to) overlook complications about theta QCD
- But nevertheless is correct, theta QCD ~ 1 (in fact always about $\pi/2$)

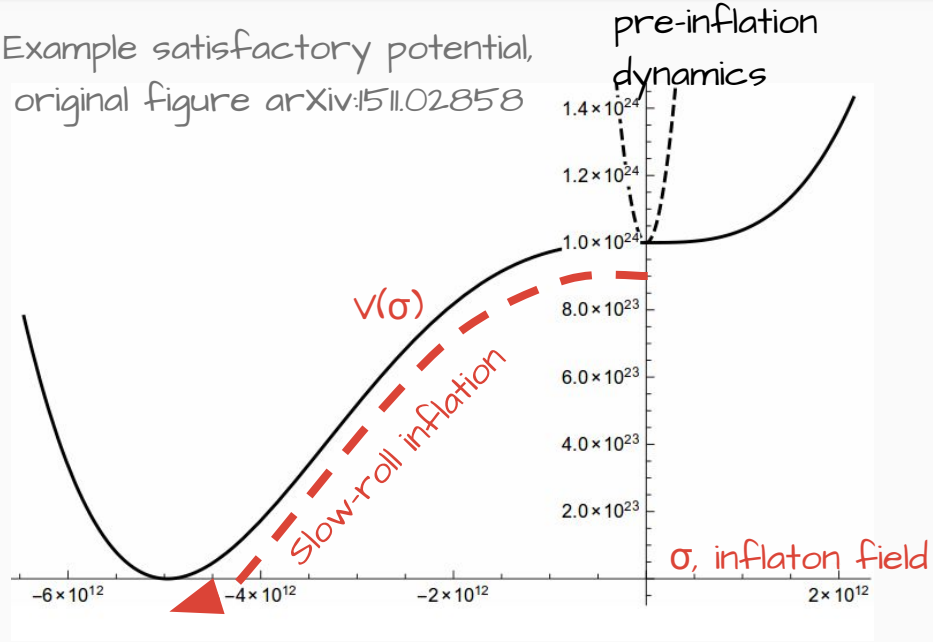
Derivations in sec. 3. in arXiv:1:602.03889

Inflation

arXiv:1511.02858
Raidal et al.

- Require >~ 50 e-folds of inflation after relaxation
- Need H <<< MP to avoid ruining barriers in potential
- Add general renormalizable single-field inflation
- Begin at origin (pre-inflation dynamics)
- Predict r, n_s etc as measured by Planck/BICEP

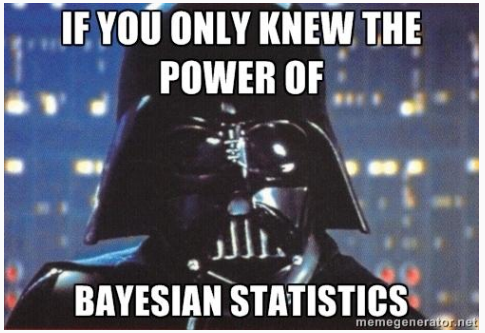
Example satisfactory potential, original figure arXiv:1511.02858



Built minimal model. Does it solve fine-tuning problem?

- Built it to solve hierarchy problem. Less fine-tuned than SM?
- **RELAXION BETTER THAN SM + SCALAR-FIELD INFLATION?**
- What does **BETTER THAN** mean?
- I will take **BETTER** to mean **MORE PLAUSIBLE**
- Calculate plausibility with **BAYESIAN STATISTICS!**

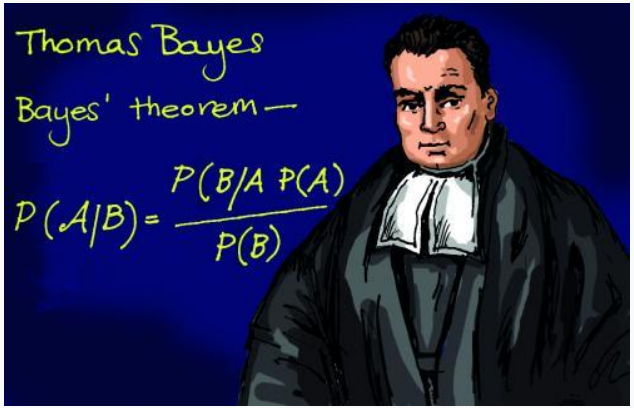
First modern discussion Jeffreys, Theory of Probability (1939) reissued OUP. Animated discussion Jaynes, Probability Theory: The Logic of Science, (2004) CUP



$$p(\text{model} \mid \text{data}) = \frac{p(\text{data} \mid \text{model}) * p(\text{model})}{p(\text{data})}$$

- Logical framework for assigning belief to theories
- Before data: Assign belief to theory (prior)
- Bayes' theorem: Update prior with data
- After data: Find most plausible model (posterior)
- Prior updated with so-called

BAYESIAN-EVIDENCE = P(DATA | MODEL)



Bayes, An Essay towards solving a Problem in the Doctrine of Chances (1763)

Bayesian evidence

Pedagogical ref: Gregory. Bayesian Logical Data Analysis for the Physical Sciences (2005), CUP

- The non-trivial calculation is **BAYESIAN-EVIDENCE**

$$p(\text{data}\,|\,\text{model}) = \int p(\text{data}\,|\,\text{model}, x) \cdot p(x\,|\,\text{model})\,\mathrm{d}x$$

Evidence

Likelihood: Planck/BICEP, MZ etc

Prior: I pick log priors

- No tricks, just not much time to explain all details
- Difficult integral! Use MC integration (nested sampling algorithm)

Skilling et al (algorithm)
Feroz et al (MultiNest)

Automatic Occam's razor

Exposition in HEP, see e.g. Fowlie, Trotta et al, Allanach et al, Fichet, and refs therein. General idea: see e.g. MacKay Bayesian Methods for Adaptive Models (1991), Jeffereys & Burger Sharpening Occam's Razor on a Bayesian Strop (1991)

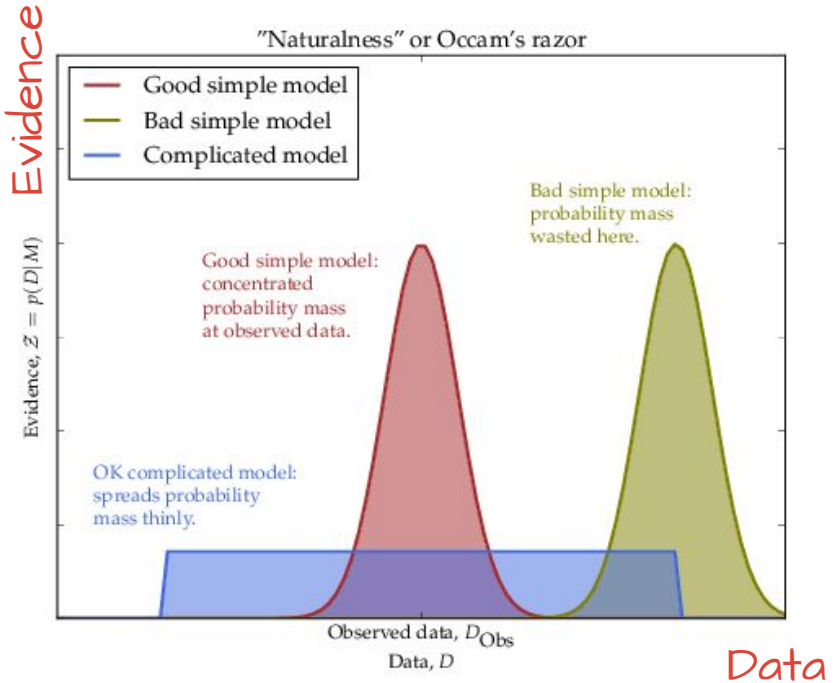
- Automatic penalty for fine-tuning/naturalness/Occam's razor
- Nothing added by hand or in priors. No tricks. Just not time for details
- Formalizes fine-tuning/naturalness - correct ideas that lacked logical framework
- A model that makes a wrong generic prediction is implausible (compared to another model)
- SM generic prediction **WEAK SCALE ~ PLANCK SCALE**

This is a result of applying Bayes. Not an extra principle

Bayesian version of Occam's razor

Fig. 1 of 1403.3407. Original Fig. MacKay Bayesian Methods for Adaptive Models (1991)

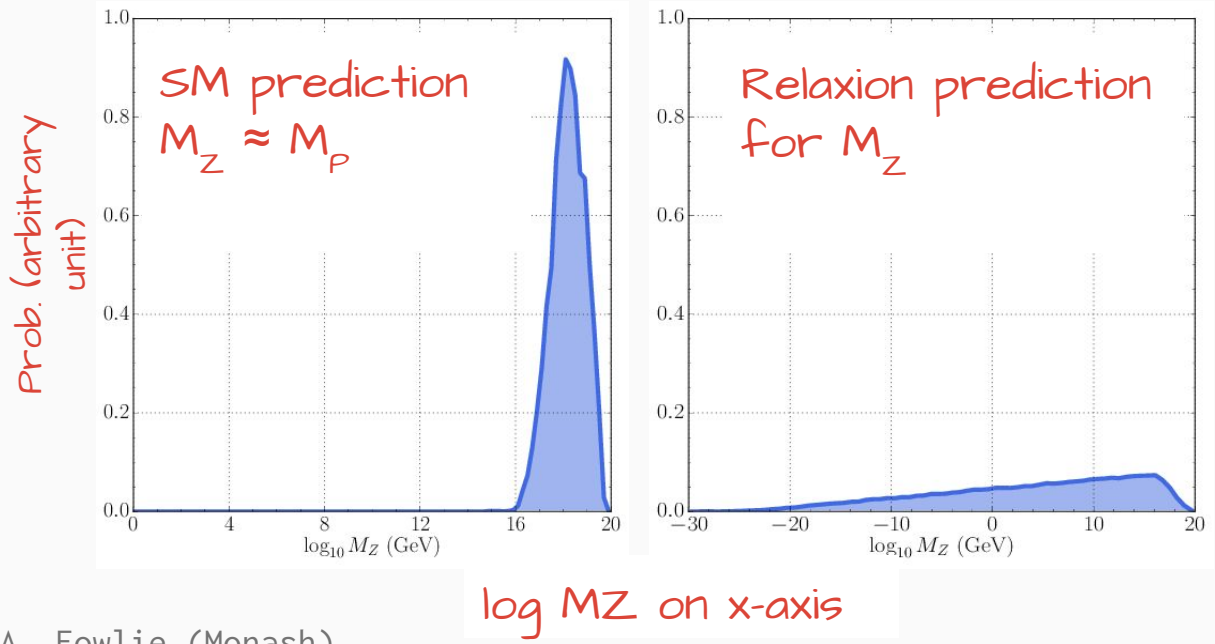
- Evidence is a pdf as function of data
- Total prob. = areas of plots = 1
- Good model spends probability mass at observation
- Bad model squanders probability mass away from observed data
- That's it. Fine-tuning/naturalness etc. Nothing added ad hoc
- Different people mean different things by Occam's razor. This doesn't justify most of that stuff



Finally: test relaxion model vs. SM with Bayes

My language
SM = SM with
Planck-scale
quadratic
corrections

Looking only at Z mass i.e. weak scale



- Relaxion prediction for Z mass broader (more complicated model)
- But higher probability density at correct scale!
- SM is simple model
- But simple prediction is wrong!

Considering Z mass alone, relaxion model solves hierarchy problem. Favoured by big Bayes-factor. But adding other things...

Is relaxion natural?

i.e. with Bayes in mind, is relaxion more plausible than SM?

- **We found, all data considered, relaxion much worse than SM!**
- Our Bayesian-evidences reveal that relaxion favored by weak scale
- But preference destroyed by fine-tuned inflation
- And constraints on Hubble parameter during relaxation
- Final result: SM + scalar-field inflation 1E25 times more plausible than relaxion model (after seeing all data)!
- Hierarchies introduce enormous factors in Bayes-factors

Relaxion needs low-scale inflation. New hierarchy problem

See also Jaeckel et al 1508.03321, Raidal et al 1511.02858 for further discussions of FT in relaxion models

Summary: minimal relaxion models not natural

1. Simple analytic formulae for relaxion model (which claims to solve fine-tuning problem)
2. First statistical analysis of relaxion model
3. Bayesian statistics includes automatic penalties for fine-tuning/naturalness
4. Found that, all told, relaxion models were much worse than SM + single-field inflation
5. Problems with unusual cosmology