Origins of Parameters in Adimensional Models

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Overview

Renormalization & adimensional models

Renormalization

Nutshell

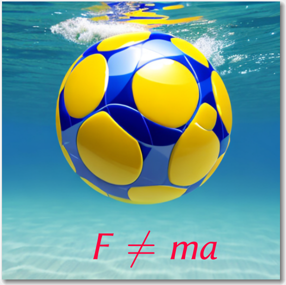
- Physical theory with some parameters λ — describing some phenomena
- "Conditions" — temperature, electric field, magnetic field, pressure — change
- Do we need a new theory?
- Not necessarily. Renormalize parameters λ — adjust them but keep same theory

Ball in water

- Consider a ball of mass m and volume V. Acceleration follows Newton's second law

$$F = ma$$

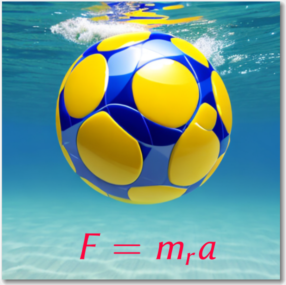
- Now put the ball under water (fluid of density ρ)
- Does Newton's law hold?



Ball in water

- To accelerate the ball, you need to move the ball and the water in front of it
- Newton's law holds if you just renormalize the mass when we change the density

$$m_r = m + \frac{1}{2} V \rho$$



Quantum Field Theory

Nutshell

- Quantum Field Theory (QFT) — theory of fundamental particles
- Governs how particles behave — particle masses and interaction strengths
- Combines special relativity and quantum mechanics
- Experimentally tested to extraordinary precision

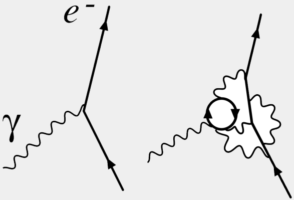
Renormalization in QFT

Nutshell

- Just like the ball in water
- The way particles interact depends on the characteristic energy of the interaction
- If you want to predict physics at a different energy, don't throw out the theory
- Keep the theory, but renormalize the parameters as functions of energy

Renormalization group equations

- Particles can interact with a cloud of virtual particles



- At larger energies, more loops of particles are relevant
- Parameter dependence on energy governed by differential equations — reormalization group equations — e.g.,

$$\frac{d\alpha_s}{d \ln Q} = -\beta_0 \alpha_s^2 \quad \text{where} \quad \beta_0 > 0$$

Adimensional models

- No fundamental dimensional constants in nature [1]
- No exotic physics or strings — just QFT
- All scales generated by quantum effects
- For example, consider Newtonian gravity between two masses

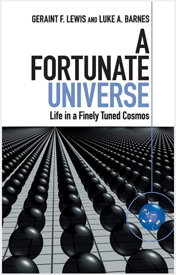
$$F = G\frac{m_1 m_2}{r^2}$$

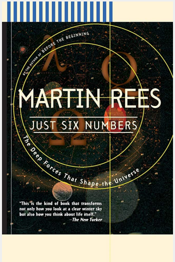
$$G = 6.67 \times 10^{-11}\, \mathrm{m^3 kg^{-1} s^{-2}}$$

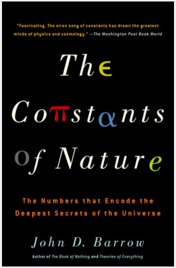
- Newton's constant G is dimensional
- Must be generated by quantum effects

ORIGINS OF FUNDAMENTAL PARAMETERS

- Adimensional models say there are no fundamental dimensional constants
- And nothing exotic or dramatic beyond QFT — just QFT
- What is the origin of the fundamental parameters?







Renormalization group invariant distributions

Are they random?

- Perhaps fundamental parameters are randomly chosen
- From what distribution?
- If there are no fundamental scales, that distribution had better not depend on energy
- Can that work? The parameters depend on energy!
- Require distributions that don't refer to any particular dimensional scale or energy

Changing variables

- The densities for a parameter $y = f(x)$ and x are connected by

$$p_Y(y) = p_X(x)\ |\mathcal{J}|$$

  where $|\mathcal{J}|$ is the Jacobian for the transformation between x and y
- In this simple one-dimensional case,

$$|\mathcal{J}| = \left|\frac{dx}{dy}\right|$$

- The distribution would be invariant under this transformation if p_X and p_Y were the same function [2, 3],

$$p \equiv p_Y = p_X$$

Changing variables

Example — scale parameter

- Consider the reals and the transformation $x \to Ax$
- The invariant measure is

$$\mu([x, y]) = \log y - \log x$$

- The invariant distribution is

$$p(x) \propto \frac{1}{x}$$

- or equivalently,

$$p(\log x) \propto \text{const.}$$

Changing variables

Example — invariants

- Suppose that we considered dependent re-scalings for two parameters, $x \to Ax$ and $y \to Ay$
- Invariance cannot uniquely dictate the form of the two-dimensional prior, as it could be

$$p(x, y) \propto \frac{f(x/y)}{xy}$$

- x/y is a group invariant
- The function f isn't restricted, though it must satisfy

$$\int f(z)dz = 1$$

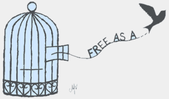
RG invariant distributions

- We want to construct distributions for an adimensional theory's parameters that don't refer to any particular energy scale
- We must construct distributions that are invariant under the renormalization group evolution
- If we succeed, fundamental parameters could be random draws from these specific distributions
- If we fail, the parameters must originate in some other way

Example

Total Asymptotic Freedom

- We consider a totally asymptotically free QFT [4]
- This means that no parameters blow up at finite-time when we increase the energy
- Our model contains two particles and two dimensionless parameters — α and λ
- They change how strongly particles interact



Total Asymptotic Freedom

- The parameters depend on energy
- The dependence governed by the renormalization group equations

$$\frac{d\lambda}{d\ln Q} = s_\lambda \lambda^2 - s_{\lambda g}\lambda g^2 + s_g g^4$$
$$\frac{d\alpha_s}{d\ln Q} = -\beta_0 \alpha_s^2$$

  where the coefficients $s > 0$ and $\beta_0 > 0$
- Let's consider the solutions to these coupled differential equations

RG flow

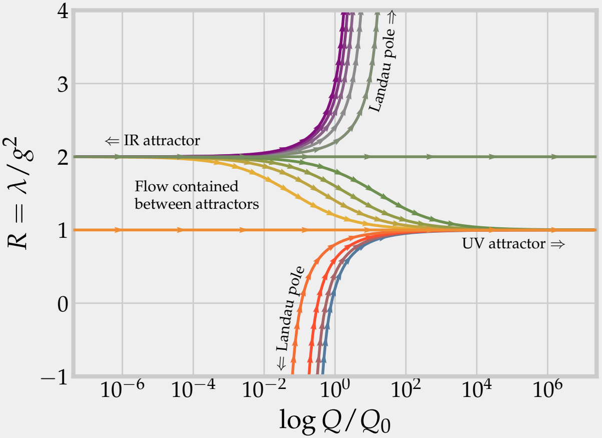


Figure: Flow contained between attractors

RG flow

- The attractors are at R_{IR} and R_{UV}
- The couplings flow according to

$$\begin{aligned} R(Q) &\equiv \frac{\lambda(Q)}{4\pi\alpha(Q)} \\ &= R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}})\,\frac{1}{2}\left[1 - \tanh\left(C \ln \alpha(Q) + \Theta\right)\right] \end{aligned}$$

  with constants C and Θ
- Θ is an RG invariant

$$\frac{d\Theta}{dQ} = 0$$

- This parameter controls the initial value of $R(Q)$ and may be written as a function of the parameters, $\Theta(R, \alpha)$

RG flow in simple TAF model

- We compute the Jacobian for a transformation between two energy scales
- With it, we find an RG invariant measure on $(R_{\text{UV}}, R_{\text{IR}})$,

$$p(R \,|\, \alpha) \propto \frac{f(\Theta(R, \alpha))}{(R_{\text{IR}} - R)\,(R - R_{\text{UV}})}$$

$$p(\alpha) \propto \frac{1}{\alpha^2}$$

- Conditional distribution $p(R \,|\, \alpha)$ has poles at attractors
- The factor $f(\Theta(R, \alpha))$ is arbitrary — unconstrained because Θ is RG invariant
- Same form at every scale; though shape of distributions flows as α flows

Role of $f(\Theta)$

Controls flow

- The function $f(\Theta)$ must controls the flow of probability from R_{IR} to R_{UV} as we increase energy

Now, as an example, consider a standard normal, $f(\Theta) = \mathcal{N}(0, 1)$ with $R_{\text{IR}} = 2$ and $R_{\text{UV}} = 1$

Measure flows between IR and UV attractor

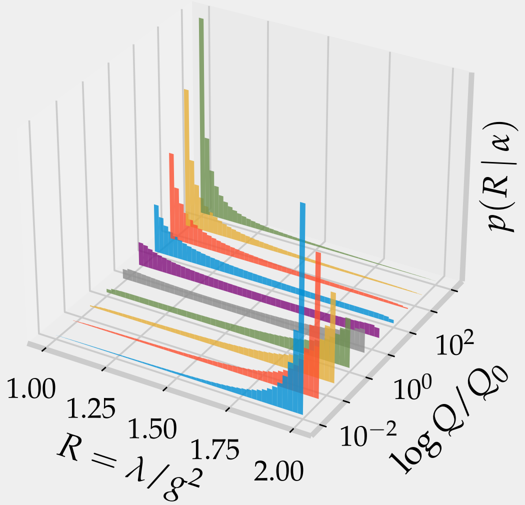


Figure: The measure moves the probability mass between the attractors

Conclusions

- Adimensional models — no fundamental dimensional parameters
- No exotic or dramatic physics beyond what we know
- Can we say anything about the origins of the dimensionless parameters?
- Yes we can, because they depend on energy
- If they are random draws, they must originate from an invariant distribution of the renormalization group
- If invariant distribution doesn't exist, they cannot originate as random draws
- Sheds light on deep question — origins of fundamental parameters

References I

[1] A. Salvio and A. Strumia, *Agravity*, *JHEP* **06** (2014) 080 [1403.4226].

[2] J. Hartigan, *Invariant Prior Distributions*, *The Annals of Mathematical Statistics* **35** (1964) 836 .

[3] E.T. Jaynes, *Prior probabilities*, *IEEE Transactions on Systems Science and Cybernetics* **4** (1968) 227, URL.

[4] G.F. Giudice, G. Isidori, A. Salvio and A. Strumia, *Softened Gravity and the Extension of the Standard Model up to Infinite Energy*, *JHEP* **02** (2015) 137 [1412.2769].