Origins of parameters in adimensional models

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Overview

Adimensional models

In a nutshell

- We observe $m_H^2 \lll M_{\text{UV}}^2$
- Weak scale radiatively unstable — unprotected by symmetries
- Expect theoretically $\Delta m_H^2 \sim M_{\text{UV}}^2$
- Fine-tuning between m_H^2 and Δm_H^2 — ugly, unnatural and moreover implausible

In a nutshell

- Protect it by symmetries — supersymmetry
- Lower the UV scales — large extra dimensions, bring the Planck scale to the weak scale!
- No fundamental scalars — technicolor
- Relaxation — strange dynamics that trap weak scale, perhaps near QCD scale — relaxion
- New physics near the TeV scale

In a nutshell

- Hang on. Have you seen the LHC results?
- No supersymmetry. No large extra dimensions. No signatures of naturalness near the TeV scale
- Maybe there are no UV scales [1–5]
- Maybe the top really is the top

Traditional picture

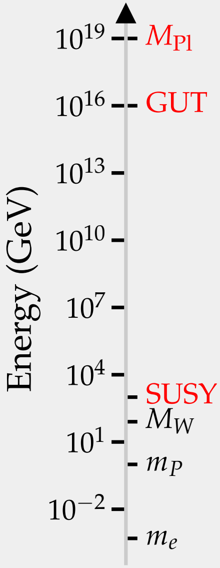


Figure: Succession of massive scales and new physics. Hierarchy problems.

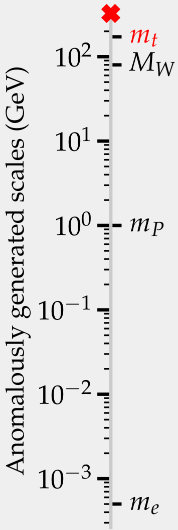


Figure: The top is the top. No classical scales. Everything generated anomalously.

- No classical scales at all — classically scale invariant
- All scales generated anomalously through dimensional transmutation
- This could be by the Coleman-Weinberg mechanism or by confinement
- No quadratic corrections to the weak scale

$$\Delta m_H^2 = 0$$

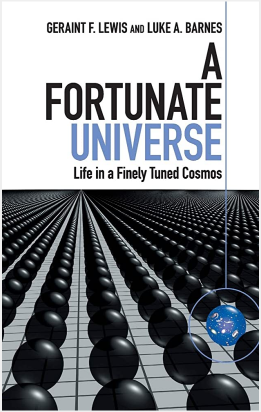
on dimensional grounds — nothing to put on the right-hand side [6]

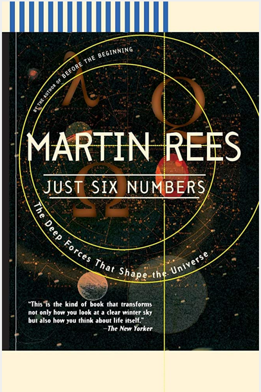
Can that really work?

What about gravity?

- Maybe gravity fits inside this idea
- Quadratic gravity [7, 8] is adimensional and renormalizable
- Adimensional model could address gravity, hierarchy problem, and other shortcomings of the Standard Model, including inflation [9]
- Suffers from Ostrodradsky instability or ghosts, though may be viable [10–14]
- No easy path to quantum gravity; must consider paths with obstacles

- Deep question — what is the origin of our fundamental parameters?
- Adimensional models posit no new dimensional physics
- Does it close door for explanations of fundamental parameters?





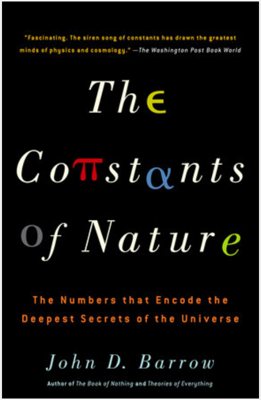


Figure: Explored in popular science but rarely in `hep-ph`

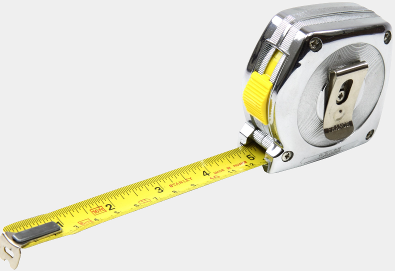
Random?

Are they random?

- Perhaps they're randomly chosen
- From what distribution?
- If there are no fundamental scales, that distribution had better not depend on any scale
- We want to construct distributions for an adimensional theory's dimensionless parameters that don't refer to any particular dimensional scales

- If there are fundamental scales, it could be scale dependent
- In grand unified theories (GUTs), for example, we might assume there is a unification scale, M_X
- We could write distributions for a unified coupling at that scale, $Q \approx M_X$
- They would look different at other scales, but that's alright as M_X is special

Invariant measures



- The densities for a parameter $y = f(x)$ and x are connected by

$$p_Y(y) = p_X(x)\ |\mathcal{J}|$$

where $|\mathcal{J}|$ is the Jacobian for the transformation between x and y

- In this simple one-dimensional case,

$$|\mathcal{J}| = \left|\frac{dx}{dy}\right|$$

- The distribution would be invariant under this transformation if p_X and p_Y were the same function,

$$p \equiv p_Y = p_X$$

- Formally, $p(x)dx$ would be an invariant measure (see e.g., ref. [15–18])

Measure theoretic

- Measure — assigns non-negative value to subsets of a space
- Must satisify

  $$\mu(\phi) = 0 \quad \text{and} \quad \mu(\cup_i A_i) = \sum_i \mu(A_i)$$

  for disjoint sets A_i
- Further conditions for a probability measure, e.g., probability on A satisfies $\mu(A) = 1$ and A is a σ-algebra

Topological group

- The parameter transformations may form a topological group
- Requires that transformations are continuous, associative and closed, and the existence of an inverse and an identity
- The topological space may be locally compact — no holes or spikes — in which case the group is said to be locally compact
- The real numbers — topological group under addition and locally compact as no holes
- The group parameters may be defined on a closed interval, in which case the group is said to be compact
- Lorentz group — defined on $[0, c)$ — not compact

- A (right) invariant measure satisfies

$$\mu(S) = \mu(Sg)$$

  for every subset S and every group element g
- This invariant measure is the (right) Haar measure of the group
- Haar measure natural notion of volume

Existence of Haar measure

- Haar measure exists for any locally compact group
- Though proper — $\mu(G) < \infty$ — if and only if group is compact

- Consider the reals and the transformation $x \to x + A$
- The invariant measure is the Lebesgue measure

$$\mu([x, y]) = x - y$$

- The invariant distribution $p(x)dx \propto \mu(dx)$ is simply

$$p(x) \propto \text{const.}$$

- Consider the reals and the transformation $x \to Ax$
- The invariant measure is more involved

$$\mu([x, y]) = \log y - \log x$$

- The invariant distribution is

$$p(x) \propto \frac{1}{x}$$

- or equivalently,

$$p(\log x) \propto \text{const.}$$

Impropriety

- The scale and shift invariant distributions are examples of improper distributions
- They weren't compact spaces
- They cannot be normalised to one because

$$\int p(x)dx = \infty$$

- We cannot sample from them
- Not useless, however, as improper prior + likelihood may lead to a proper posterior

$$\int p(K \mid x)p(x)dx < \infty$$

Invariants

- The number of distributions that can be found by a group invariance is equal to the size of the invariance group, n
- We can re-parameterise our d dimensional space as n parameters and $d - n$ group invariants
- For example, rotations in d dimensions — an invariant radius and $d - 1$ angles
- The measures for the invariants are arbitrary as they do not transform under the group
- $|\mathcal{J}| = 1$ for the invariants

- Suppose that we considered dependent re-scalings for two parameters, $x \to Ax$ and $y \to Ay$
- Invariance cannot uniquely dictate the form of the two-dimensional prior, as it could be

$$p(x, y) \propto \frac{f(x/y)}{xy}$$

- x/y is a group invariant
- The function f isn't restricted, though it must satisfy

$$\int f(z)dz = 1$$

Examples

- Similarly, if we were to consider dependent shifts, $x \to x + A$ and $y \to y + A$, we would obtain

$$p(x, y) \propto f(x - y)$$

- $x - y$ is a group invariant
- The function f isn't restricted, though it must satisfy

$$\int f(z)\,dz = 1$$

- We want to construct distributions for an adimensional theory's parameters that don't refer to any particular dimensional scale
- We must consider the RG evolution of the parameters and build the RG invariant measures and distributions
- If we succeed, fundamental parameters could be draws from these distributions
- If we fail, the parameters must originate in some other way

QCD

QCD β-function

- The β-function in QCD

$$\frac{d\alpha_s}{d\ln Q} = -\beta_0 \alpha_s^2 \quad \text{where} \quad \beta_0 > 0$$

- Solving this differential equation yields

$$\alpha_s(Q) = \frac{\alpha_s(Q')}{1 - \alpha_s(Q')\,\beta_0 \ln\left(Q'/Q\right)}$$

- This may be re-written as

$$\alpha_s(Q) = \frac{1}{\beta_0 \ln\left(Q/\Lambda_{\mathrm{QCD}}\right)}$$

- This used the RG-invariant QCD scale

$$\Lambda_{\mathrm{QCD}} = Q e^{-\frac{1}{\beta_0 \alpha_s(Q)}}$$

QCD RG flow

- The coupling flows to $\alpha_S = 0$ — asymptotic freedom
- This is a UV attractor
- Landau-pole — finite-time blow up — in the IR at the QCD scale
- RG evolution through Landau pole impossible

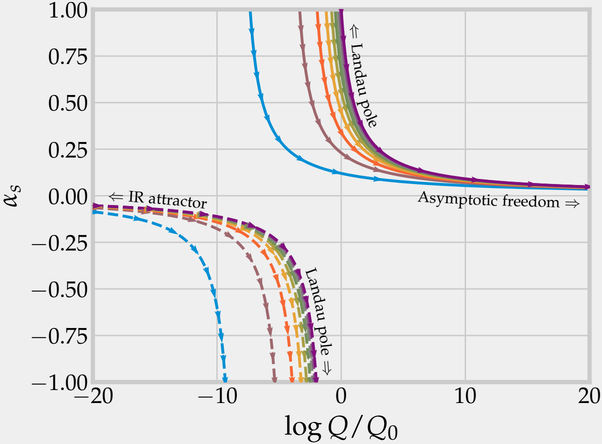


Figure: RG flow in QCD.

- We cannot evolve through Landau pole
- The RG transformations aren't closed — as evolving to $Q < \Lambda_{\text{QCD}}$ undefined
- This means that an invariant measure cannot exist
- For illustrative purposes, suppose

$$\alpha_s(Q) = \frac{1}{\beta_0 \ln\left(Q/\Lambda_{\text{QCD}}\right)}$$

  valid for any Q
- The Jacobian for flowing from $Q \to Q'$

$$|\mathcal{J}| = \left|\frac{d\alpha_s(Q')}{d\alpha_s(Q)}\right| = \left|\frac{\alpha_s(Q')}{\alpha_s(Q)}\right|^2$$

Invariant measure

- The Jacobian rule leads to

$$p(\alpha_s(Q)) = p'(\alpha_s(Q'))\,|\mathcal{J}|$$

- Requiring that $p = p'$ gives an invariant distribution

$$p(\alpha_s) \propto \frac{1}{\alpha_s^2}$$

- Wonderful! but improper!

$$\int_0 \frac{d\alpha}{\alpha^2} = \infty$$

Why is it improper, physically?

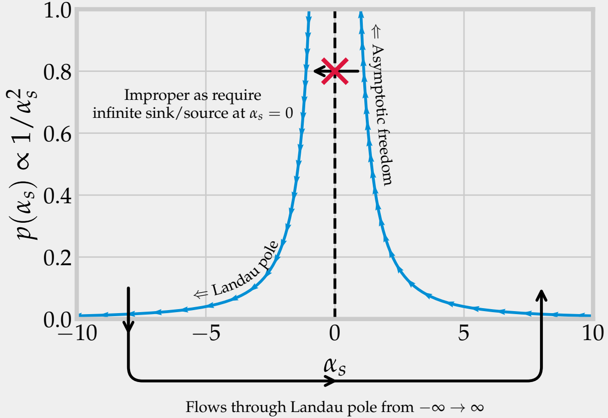


Figure: We cannot cross from $+0$ to -0

- The invariant measure for the QCD coupling leads to

$$p(\log \Lambda_{\text{QCD}}) = \text{const.}$$

- This is a scale invariant distribution — invariant under

$$\Lambda_{\text{QCD}} \to A\Lambda_{\text{QCD}}$$

- Intuitive — how could there be any preferred QCD scale in the theory?
- Improper — having no preferred scale means that it is improper

Why is it improper, group theoretically?

- The group wasn't compact; specifically, as we omitted $\alpha = 0$
- $\alpha = 0$ trivial; stays zero forever
- If included, the invariant measure is a Dirac mass at zero, $\delta(\alpha)$

Does it matter?

- In our physics education, we have a series of shocks when we discover seeming important quantities can be set arbitrarily, e.g.,

$$c = \hbar = G = 1$$

- We can use Λ_{QCD} to define a system of units

$$\Lambda_{\mathrm{QCD}} = 1$$

- Measure any other scales relative to QCD scale

Coleman-Weinberg Model

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Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*

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(Received 8 November 1972)

We investigate the possibility that radiative corrections may produce spontaneous symmetry breakdown in theories for which the semiclassical (tree) approximation does not indicate such breakdown. The simplest model in which this phenomenon occurs is the electrodynamics of massless scalar mesons. We find (for small coupling constants) that this theory more closely resembles the theory with an imaginary mass (the Abelian Higgs model) than one with a positive mass; spontaneous symmetry breaking occurs, and the theory becomes a theory of a massive vector meson and a massive scalar meson. The scalar-to-vector mass ratio is computable as a power series in e, the electromagnetic coupling constant. We find, to lowest order, $m^2(S)/m^2(V) = (3/2\pi)(e^2/4\pi)$. We extend our analysis to non-Abelian gauge theories, and find qualitatively similar results. Our methods are also applicable to theories in which the tree approximation indicates the occurrence of spontaneous symmetry breakdown, but does not give complete information about its character. (This typically occurs when the scalar-meson part of the Lagrangian admits a greater symmetry group than the total Lagrangian.) We indicate how to use our methods in these cases.

- Scalar with a $U(1)$ gauge symmetry

$$\mathcal{L} = \left|D_\mu \phi\right|^2 - \frac{\lambda}{4!}|\phi|^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

- Tree-level scalar mass was set to zero, such that the model is adimensional
- Masses are generated by radiative symmetry breaking
- The symmetry breaking scale, v, defined as the solution for

$$\lambda(Q = v) = \frac{33}{8\pi^2}e^4(Q = v)$$

- The ratio of the scalar and vector mass predicted

$$\frac{m_\phi^2}{m_V^2} = \frac{2}{3\pi}\alpha(v)$$

- RG equations at one-loop are [19–21]

$$\frac{d\lambda}{d\ln Q} = \frac{10}{3(4\pi)^2}\lambda^2 - \frac{12}{(4\pi)}\alpha\lambda + 36\alpha^2$$
$$\frac{d\alpha}{d\ln Q} = -\beta_0\alpha^2$$

- For the gauge coupling, this time $\beta_0 < 0$ though otherwise identical as before

- There are exact solutions

$$\alpha(Q) = \frac{\alpha(Q')}{1 - \alpha(Q')\,\beta_0 \ln\left(Q'/Q\right)}$$

$$\lambda(Q) = \gamma\alpha(Q) \tan\left[\sqrt{719}\ln\alpha(Q) + \Theta\right] + \delta\alpha(Q)$$

where the coefficients are

$$\gamma = \frac{\sqrt{719}}{10}4\pi \quad \text{and} \quad \delta = \frac{19}{10}4\pi$$

- The quantity Θ may be chosen to fix $\lambda(Q')$ through

$$\Theta = \arctan\left[\frac{\lambda(Q') - \delta\alpha(Q')}{\gamma\alpha(Q')}\right] - \sqrt{719}\ln\alpha(Q')$$

This parameter is in fact an RG invariant

RG flow in CW model

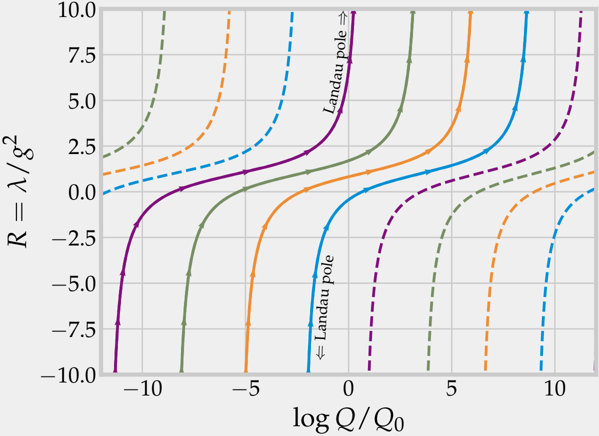


Figure: The tangent swings rapidly between Landau poles

Invariant measure in CW model

- As before, make RG transformations closed by treating the RG solutions as valid at any Q
- Compute the Jacobian of the transformations

$$|\mathcal{J}| = \frac{\alpha'}{\alpha} \frac{\alpha'^2 + (\lambda' - \delta\alpha')^2}{\alpha^2 + (\lambda - \delta\alpha)^2}$$

- Thus the distributions are related by

$$p(\lambda(Q), \alpha(Q)) = p'(\lambda(Q'), \alpha(Q'))\,|\mathcal{J}|$$

- We construct an RG invariant distributions by requiring that p and p' are the same function,

$$p(\lambda, \alpha) \propto \frac{f(\Theta)}{\alpha\left[\gamma\alpha^2 + (\lambda - \delta\alpha)^2\right]}$$

- We may marginalise λ, finding,

$$p(\lambda \mid \alpha) = \frac{f(\Theta)}{\gamma\alpha\pi\left[1 + \left(\frac{\lambda - \delta\alpha}{\gamma\alpha}\right)^2\right]},$$

$$p(\alpha) \propto \frac{1}{\alpha^2},$$

- The same improper measure for α
- The conditional measure for λ is proper, and resembles a Cauchy distribution located at $\delta\alpha$ with scale parameter $\gamma\alpha$
- The factor $f(\Theta)$ arbitrary though must satisfy

$$\int f(\Theta) d\Theta = 1$$

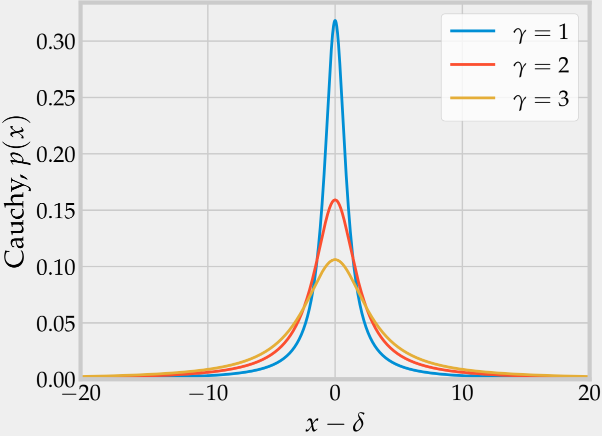
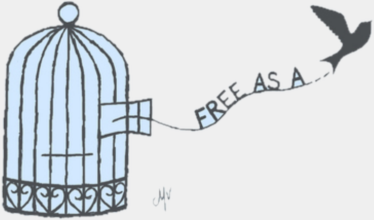


Figure: Cauchy distribution

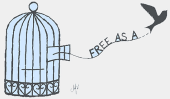
- Landau poles mean that RG evolution isn't closed
- To construct the invariant measure, we assumed RG evolution 'through' Landau poles
- Hmm.
- Even accepting that, the results are practically useless
- Knowing the parameters α and λ at a scale Q isn't enough
- We need to know 'domain' where theory valid — valid between which two Landau poles?

Total Asymptotic Freedom



Total Asymptotic Freedom

- QCD was trivial — $\Lambda_{\text{QCD}} = 1$
- Scalar QED suffered from Landau poles in the IR and UV
- Consider theory with no Landau poles in UV — easiest theories are totally asymptotically free theories [22]
- Make a simple model — a scalar with a non-Abelian gauge interaction
- Leave it general — don't specify group or particle content



Total Asymptotic Freedom

- Form of RG equation for the gauge coupling identical to that in QCD — no Landau pole in UV
- Agnostic about form of quartic RG equation;

$$\frac{d\lambda}{d \ln Q} = s_\lambda \lambda^2 - s_{\lambda g} \lambda g^2 + s_g g^4$$

- In known QFT, coefficients $s > 0$

RG flow in simple TAF model

- Convenient to define

$$C \equiv \frac{s_g}{\beta_0}$$

$$D \equiv \frac{s_{\lambda g} - \beta_0}{2 s_g}$$

$$E \equiv D^2 - \frac{s_\lambda}{s_g}$$

- For $E = 0$, trivial fixed-flow
- For $E < 0$, Coleman-Weinberg type behaviour — tangent that swings rapidly between Landau poles
- For $E > 0$, may avoid Landau poles

RG flow in simple TAF model

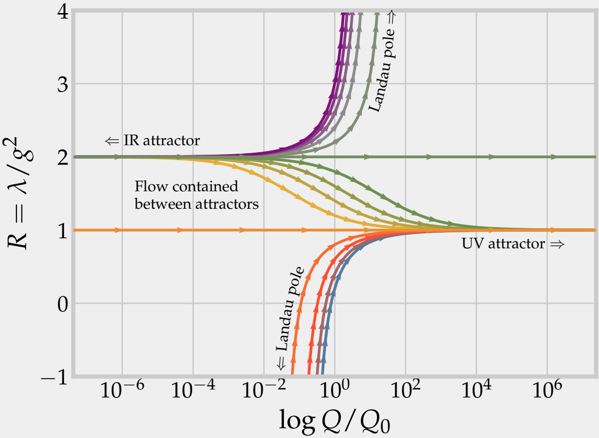


Figure: Flow contained between attractors

RG flow in simple TAF model

- The attractors are at

$$R_{\mathrm{IR}} = \frac{1}{D - \sqrt{E}} \quad \text{and} \quad R_{\mathrm{UV}} = \frac{1}{D + \sqrt{E}}.$$

- If at any scale the ratio lies inside $[R_{\mathrm{UV}}, R_{\mathrm{IR}}]$, it stays trapped inside that interval

$$\begin{aligned} R(Q) &\equiv \frac{\lambda(Q)}{4\pi\alpha(Q)} \\ &= R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}})\,\frac{1}{2}\left[1 - \tanh\left(C\sqrt{E}\ln\alpha(Q) + \Theta\right)\right] \end{aligned}$$

- $R = R_{\mathrm{UV}}$ and $R = R_{\mathrm{IR}}$ are special fixed flows
- Otherwise, it flows to a Landau pole in the IR or UV

RG flow in simple TAF model

- Inside the solution

$$R(Q) = R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}})\, \frac{1}{2}\left[1 - \tanh\left(C\sqrt{E}\ln\alpha(Q) + \Theta\right)\right]$$

  the red factor goes from 0 in the IR to 1 in the UV
- Θ is an RG invariant,

$$\Theta = \operatorname{arctanh}\left[1 - 2\left(\frac{R(Q) - R_{\mathrm{IR}}}{R_{\mathrm{UV}} - R_{\mathrm{IR}}}\right)\right] - C\sqrt{E}\log\alpha(Q)$$

- It controls $R(Q')$

RG flow in simple TAF model

- Computing the Jacobian, we find an RG invariant measure on $(R_{\mathrm{UV}}, R_{\mathrm{IR}})$,

$$
\begin{aligned}
p(R \,|\, \alpha) &\propto \frac{f(\Theta)}{(R_{\mathrm{IR}} - R)\,(R - R_{\mathrm{UV}})} \\
p(\alpha) &\propto \frac{1}{\alpha^2}
\end{aligned}
$$

- Conditional distribution $p(R \,|\, \alpha)$ has poles at attractors
- Proper so long as $f(\Theta)$ is proper
- Same form at every scale; though shape of distributions flows as α flows

Role of $f(\Theta)$

- $\Theta \equiv \Theta(R, \alpha)$ — invariant though function of R and α
- The function $f(\Theta)$ must satisfy

$$\int f(\Theta)\, d\Theta = 1$$

and thus must satisfy

$$\lim_{|\Theta| \to \infty} f(\Theta) = 0$$

- Consider behaviour of

$$R(Q) = R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}}) \frac{1}{2} \left[1 - \tanh\left(C\sqrt{E} \ln \alpha(Q) + \Theta \right) \right]$$

- In the IR where $\ln \alpha \to \infty$, $R \to R_{\mathrm{UV}}$ requires $\Theta \to -\infty$
- In the UV where $\ln \alpha \to -\infty$, $R \to R_{\mathrm{IR}}$ requires $\Theta \to \infty$

Role of $f(\Theta)$

Controls flow

- Thus the function $f(\Theta)$ must disfavour R_{UV} in the IR and R_{IR} in the UV
- Controls flow of probability from R_{IR} in the IR to R_{UV} in the UV

Now, as an example, consider a standard normal, $f(\Theta) = \mathcal{N}(0, 1)$ with $R_{\mathrm{IR}} = 2$ and $R_{\mathrm{UV}} = 1$

Measure flows between IR and UV attractor

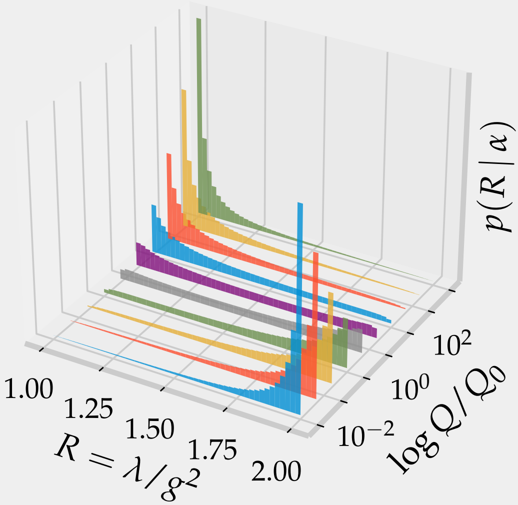


Figure: The measure moves the probability mass between the attractors

Dirac masses at attractors

- We considered $(R_{\mathrm{IR}}, R_{\mathrm{UV}})$, i.e., omitting the endpoints
- Dirac mass at the attractors would also be invariant

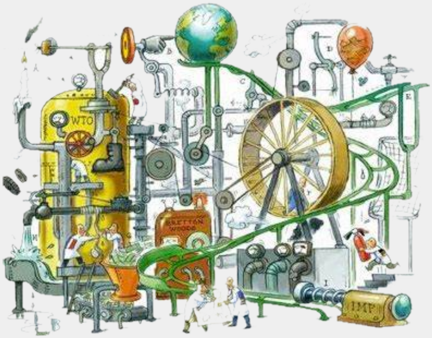
$$p(R) = \delta(R - R_{\mathrm{IR/UV}})$$

- They could be combined with our invariant distribution

What theories predict $E > 0$?

- Discussed in ref. [22]
- Certainly possible, though requires big representations and big groups
- Easier if add Yukawa interactions, though RG equations become harder to solve

Complications



- In principle, more realistic models are no different
- If RG flows forms locally compact group, invariant distribution should exist
- Though only proper if flow forms compact group
- In practice, much harder as we cannot compute the RG invariants or solve the RG equations analytically
- Maybe there are short-cuts?
- For example, Ulam's method or the ergodic hypothesis that time-averages equal space-averages

Conclusions

- Adimensional models could solve the hierarchy problem
- Possibly leads to renormalizable quadratic gravity, though suffers from ghosts
- Predict no fundamental scales at all
- If parameters originate as random draws, distributions must be scale invariant
- Explored scale invariant distributions in simple models
- This involves finding the invariant Haar measure of RG transformations
- Landau poles caused difficulties
- Successful in totally asymptotically free model

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