BAYESIAN RECONSTRUCTION OF SUSY PARAMETERS AT $\sqrt{s} = 14\,\mathrm{TeV}$ VIA THE GOLDEN DECAY

Andrew Fowlie, Malgorzata Kazana, Leszek Roszkowski

University of Sheffield
a.fowlie@sheffield.ac.uk

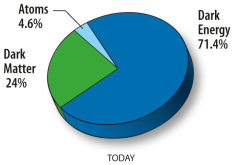
TMEX 2013

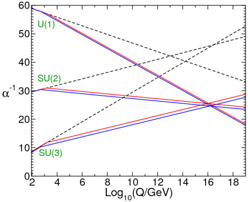
Outline

Interested in CMSSM

- SUSY motivations you have heard before. Amongst other things:

1. **Solves hierarchy problem** by cancelling divergent loops
2. **Dark matter** is lightest supersymmetric particle (*R*-parity), usually χ_1
3. **Unification of couplings** if SUSY particles included in running $\lesssim 10\,\text{TeV}$ at $\sim 10^{16}\,\text{GeV}$

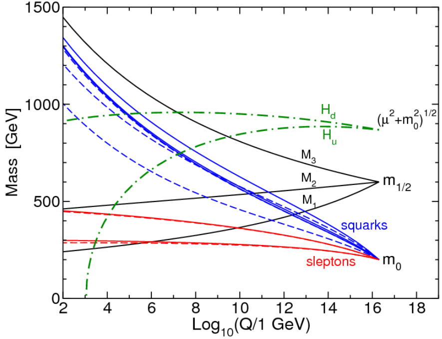




Interested in CMSSM

- CMSSM = Constrained Minimal Supersymmetric Standard Model
- Unification of MSSM soft masses at GUT scale:
  1. $m_{1/2} = M_1 = M_2 = M_3 =$ Common gaugino mass
  2. $m_0 =$ Common scalar mass
  3. $A_0 =$ Common trilinear
  4. $\tan\beta =$ Ratio of Higgs vevs
  5. sgn μ
- Run parameters to low scale with renormalisation group equations
- Calculate mass spectrum

Interested in CMSSM



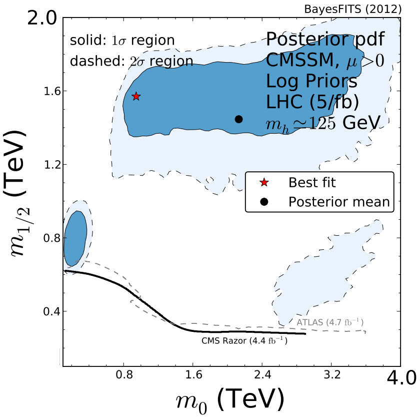
- Approximate mass relations:
  - $m_{\tilde{\chi}_1} \approx 0.4 m_{1/2}$
  - $m_{\tilde{\chi}_2} \approx 0.8 m_{1/2}$
  - $m_{\tilde{g}} \approx 2.7 m_{1/2}$
  - $m_{\tilde{\tau}_1} \approx \sqrt{0.15 m_{1/2}^2 + m_0^2}$

No SUSY so far...

- Expected light SUSY $M_{\text{SUSY}} \gtrsim M_{\text{EW}}$:
  1. 3σ from $(g-2)_\mu$ experimental hint. Light smuons?
  2. Dark matter annihilation prefers light χ_1
  3. Naturalness $\frac{\partial M_Z}{\partial M_{\text{SUSY}}}$ fine-tuning of EW scale
- Pre-LHC, CMSSM fits showed:
  - $\tilde{\chi}_i^0, \tilde{\ell} \lesssim 0.5\,\text{TeV}$
  - $\tilde{q}, \tilde{g} \lesssim 1\,\text{TeV}$.
- Ωh^2 reduced by stau-annihilation
- Nope!
- These scenarios excluded by direct searches, Higgs, etc

SUSY must be heavier...

arXiv:1206.0264



■/■ = 68%/95% region

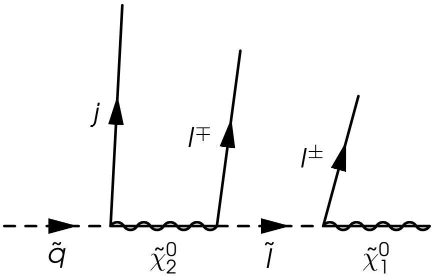
- $M_{\text{SUSY}} >$ than expected
- $m_h \approx 125$ GeV as constraining as multijet searches
- Our fits show $\lesssim$ TeV scale compatible with Higgs, etc
- Do not need $\gg 1$ TeV, split SUSY yet
- Lightest in stau coannihilation. $m_h \approx 125$ GeV with maximal mixing $M_{\text{SUSY}} \approx \sqrt{6} X_t$
- Big $\sim \pm 3$ GeV on m_h from missing higher orders

Finding heavier SUSY (1.)

- Are heavier models visible at LHC $\sqrt{s} = 14\,\text{TeV}$?
- Yes! Via a golden decay
- Can we measure the masses?
- Yes! Reconstruct sparticle masses from kinematic edges
- Preliminary golden decay studies were for light SUSY, for early LHC with $\sim 10\,\text{fb}^{-1}$
- e.g. ATLAS SU3
- Extend previous work (arXiv:1106.5117, arXiv:0907.0594)
- What might errors on SUSY masses be?
- What about resulting errors on SUSY parameters?

Finding heavier SUSY (2.)

- Famous golden decay: $\tilde{q} \to j\tilde{\chi}_2^0 \to q\ell\tilde{\ell} \to q\ell\ell\tilde{\chi}_1^0$



- Visible products:
  1. At least one jet (possible jet from initial $\tilde{g}$ decay)
  2. OSSF leptons (but near and far cannot be distinguished)
  3. MET from χ_1

Method

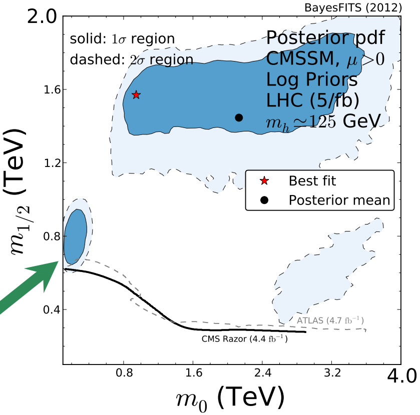
- Idea: How well might CMSSM parameters be found at 14 TeV?

1. Pick a CMSSM point allowed by experiments (e.g. m_h, Ωh^2, direct searches)
2. Monte Carlo for CMSSM point at 14 TeV
3. Simulate sparticle mass measurements from golden decay
4. Bayesian reconstruction of CMSSM parameters with simulated sparticle mass measurements

First step, picking CMSSM point (1.)

- Golden decay requires hierarchy:
  - $\tilde{q} > \tilde{\chi}_2^0 > \tilde{\ell}$
  - $\tilde{g} > \tilde{q}$ to shut $\tilde{q} \to \tilde{g} q$ spoiler
- Actually $\tilde{\chi}_2^0 > \tilde{\ell} + 50\,\mathrm{GeV}$ to avoid phase space suppression of BR
- In CMSSM, means $m_{1/2} \gtrsim m_0$
- Look again at allowed regions. Need stau-coannihilation region
- Stau-coannihilation allowed in CMSSM with golden decay

Roszkowski et al., arXiv:

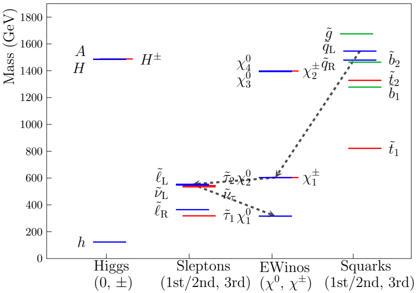


$\blacksquare / \square$ = 68%/95% region

First step, picking CMSSM point (2.)

- Local search for good point with golden decay
- Minuit to find the point with $m_{1/2} = 750\,\text{GeV}$ and:
  - $m_h \approx 125\,\text{GeV}$, within errors
  - $\Omega h^2 \approx$ WMAP/PLANCK
  - Golden decay

1. $m_{1/2} = 750\,\text{GeV}$
2. $m_0 = 230\,\text{GeV}$
3. $\tan\beta = 8.8$
4. $A_0 = -2100\,\text{GeV}$



First step, picking CMSSM point (3.)

- Important masses:

$\tilde{\chi}_1^0$	316 GeV	$\tilde{\chi}_2^0$	604 GeV
$\tilde{e}_L$	554 GeV	$\tilde{e}_R$	364 GeV
$\tilde{u}_L$	1545 GeV	$\tilde{g}$	1675 GeV
h	123.2 GeV	$\tilde{\tau}_1$	318 GeV

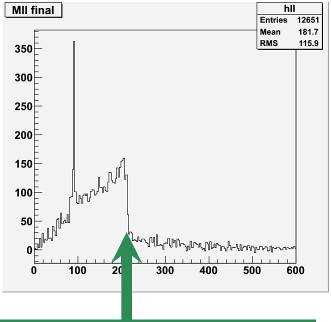
- Important observables:
  1. $\Omega h^2 = 0.11$ agreement with Planck by stau coannihilation
  2. $\mathcal{B}(B_s \to \mu^+\mu^-) = 3.2 \times 10^{-9}$ agreement with LHCb
  3. $(g-2)_\mu = 3.8 \times 10^{-10}$ poor, but so is SM
  4. Higgs reasonable agreement within theory error $m_h = 123.2 \pm 3\,\text{GeV}$

Second step, simulating golden decay

- Pythia with number of events $\Leftrightarrow \sim 100\,\text{fb}^{-1}$ at 14 TeV
- $\sim 100\,\text{fb}^{-1}$ could be collected in ~ 2 years
- Though in reality now likely to be 13 TeV...
- Our benchmark mass spectrum from SoftSUSY
- Resulting in invariant mass distributions for:
  1. lepton pair ($\ell\ell$),
  2–3. and each lepton with the jet (ℓq and $\ell\prime q$),
  4. the jet and both leptons ($\ell\ell q$),
  5. and a threshold $\ell\ell q$, with $\theta > \pi/2$ between leptons in slepton frame

Third step, recover sparticle masses (1.)

- Predict "edges" in distributions from relativistic kinematics
- **Functions of four unknown sparticle masses** in golden decay, $\tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{q}, \tilde{\ell}$
- E.g., endpoint of $\ell\ell$ invariant mass distribution
- Sawtooth shape because mediated by scalar



$$m^2_{\ell\ell,edge} = \max(p^\mu_{\ell_{near}} + p^\mu_{\ell_{far}})^2$$

- Endpoints are functions of sparticle masses (e.g., arXiv:0410303):

1. $$m^2_{\ell\ell} = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}}\right)\left(m^2_{\tilde{l}} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}}}$$
2. $$m^2_{\ell q,\,\text{near}} = \frac{\left(m^2_{\tilde{q}} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}}\right)}{m^2_{\tilde{\chi}^0_1}}$$
3. $$m^2_{\ell q,\,\text{far}} = \frac{\left(m^2_{\tilde{q}} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{l}} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}}}$$
4. $$m^2_{\ell qq} = \max\left[\frac{\left(m^2_{\tilde{q}} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{\chi}^0_2}}, \frac{\left(m^2_{\tilde{q}} - m^2_{\tilde{l}}\right)\left(m^2_{\tilde{l}} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}}}\right]$$

THIRD STEP, RECOVER SPARTICLE MASSES (3.)

- Fit unknown sparticle masses to five endpoints with Root
- Single solution for sparticle masses and statistical errors
- Errors are correlated $\Rightarrow$ covariance matrix
- Basis $(m_{\tilde{\chi}_1^0}, m_{\tilde{\ell}}, m_{\tilde{\chi}_2^0}, m_{\tilde{q}})$ in $(\mathrm{Gev})^2$:

$$C_{\text{golden decay}} = \begin{pmatrix} 1674 & 995 & 991 & 1508 \\ \cdot & 595 & 592 & 899 \\ \cdot & \cdot & 589 & 894 \\ \cdot & \cdot & \cdot & 1364 \end{pmatrix}$$

- Diagonalise matrix to find errors:

$$VC^{-1}V^T \approx \mathrm{diag}\left[(0.2\,\mathrm{GeV})^{-2}, (1.6\,\mathrm{GeV})^{-2}, (1.9\,\mathrm{GeV})^{-2}, (64.9\,\mathrm{GeV})^{-2}\right]$$

- Best-determined direction $\sim \frac{1}{\sqrt{2}}(m_{\tilde{\ell}} - m_{\tilde{\chi}_2^0})$ with $\sigma = 0.2\,\mathrm{GeV}$

Third step, recover sparticle masses (4.)

- Four eigenvectors of covariance matrix:
  1. $0.2\,\text{GeV} \Leftrightarrow \quad 0.0 \cdot m_{\tilde{\chi}_1^0} + 0.7 \cdot m_{\tilde{\ell}} - 0.7 \cdot m_{\tilde{\chi}_2^0} + 0.0 \cdot m_{\tilde{q}} \approx \frac{1}{\sqrt{2}}(m_{\tilde{\ell}} - m_{\tilde{\chi}_2^0})$
  2. $1.6\,\text{GeV} \Leftrightarrow \quad 0.2 \cdot m_{\tilde{\chi}_1^0} - 0.4 \cdot m_{\tilde{\ell}} - 0.5 \cdot m_{\tilde{\chi}_2^0} + 0.8 \cdot m_{\tilde{q}}$
  3. $1.9\,\text{GeV} \Leftrightarrow -0.8 \cdot m_{\tilde{\chi}_1^0} + 0.4 \cdot m_{\tilde{\ell}} + 0.4 \cdot m_{\tilde{\chi}_2^0} + 0.3 \cdot m_{\tilde{q}}$
  4. $64.9\,\text{GeV} \Leftrightarrow \quad 0.6 \cdot m_{\tilde{\chi}_1^0} - 0.4 \cdot m_{\tilde{\ell}} - 0.4 \cdot m_{\tilde{\chi}_2^0} - 0.6 \cdot m_{\tilde{q}}$
- Three well determined directions $\sigma \leq 2\,\text{GeV}$
- But one poor $\sigma \approx 65\,\text{GeV}$

Final step, reconstruct CMSSM parameters (1.)

- Now have all the ingredients
- Try to recover original CMSSM parameters from simulated sparticle mass measurements
- Use Bayesian statistics. Bayes theorem:

$$\underbrace{p\left(m_0, m_{1/2}, \tan\beta, A_0 | \mathbf{D}\right)}_{\text{Posterior density}} \propto \underbrace{\mathcal{L}\left(\mathbf{D} | m_0, m_{1/2}, \ldots\right)}_{\text{Likelihood}} \times \underbrace{\pi\left(m_0, m_{1/2}, \ldots\right)}_{\text{Prior}}$$

- We want to find posterior density for CMSSM, given golden decay measurements
- Marginalise posterior, to remove parameter dependencies, e.g., $p\left(m_0, m_{1/2} | \mathbf{D}\right) = \int p\left(m_0, m_{1/2}, \tan\beta, A_0 | \mathbf{D}\right) \, dA_0 \, d\tan\beta$
- Find "credible regions:" Smallest region A such that $\int_A p\left(m_0, m_{1/2} | \mathbf{D}\right) \, dm_0 \, dm_{1/2} = 95\%$

FINAL STEP, RECONSTRUCT CMSSM PARAMETERS (2.)

- Priors reflect "prior belief" in parameter space
- Choose flat priors, expect prior independence
- Likelihood $\mathcal{L}$ is a multivariate Gaussian from our golden decay simulations,

$$\mathcal{L}_{\text{golden decay}} = \exp\left[-\frac{1}{2}(M - M_{\text{benchmark}})C^{-1}(M - M_{\text{benchmark}})^T\right]$$

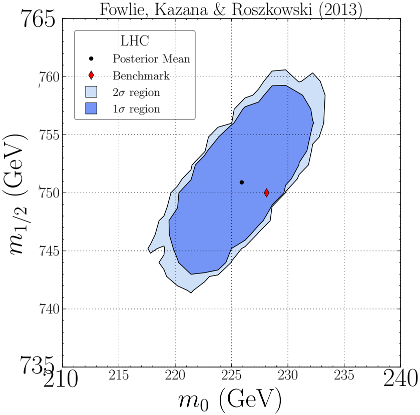
- $M = (m_{\tilde{\chi}_1^0}, m_{\tilde{\ell}}, m_{\tilde{\chi}_2^0}, m_{\tilde{q}})$ is function of $m_0, m_{1/2}, \ldots$ and C is covariance matrix from our MC
- Also apply Gaussian likelihoods for $\Omega h^2 = 0.1186 \pm 0.0031 \pm 10\%$ and $m_h = 125.8 \pm 0.5 \pm 3\,\text{GeV}$
- Supply priors and likelihoods to MultiNest. Returns posterior after a few days

Recap of method

1. Assume SUSY CMSSM benchmark point is "true"
2. Assume sparticle masses measured by golden decay at LHC $\sqrt{s} = 14\,\text{TeV}$
3. Find expected errors (covariance matrix) from MC
4. Assume flat priors for CMSSM parameters m_0, $m_{1/2}$, A_0, $\tan\beta$
5. Fit CMSSM to golden decay measurements with Bayesian statistics
6. How well do we recover the original benchmark parameters?
7. Afterwards, add information from m_h and Ωh^2 to see how much it improves recovery

Results, golden decay only — $(m_0, m_{1/2})$

$(m_0, m_{1/2})$ for $\mathcal{L}_{\text{golden decay}}$

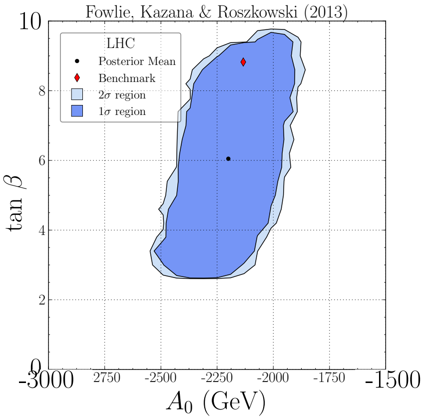


- ■/■ = 68%/95% region
  ♦/• = Benchmark/estimate
- Single correct solution found
- With this information alone, successfully recover "true" benchmark point
- Major axis ⇔ poorly measured combination of sparticle masses
- Bias for smaller m_0 and larger $m_{1/2}$

Results, golden decay only — (A_0,tan β)

- ■/■ = 68%/95% region
  ♦/• = Benchmark/estimate
- Single correct solution found
- A_0 reconstruction much poorer, ~ 0.5 TeV at 95%
- tan β determined to within a few units
- Slight positive correlation, not much though
- Bias for smaller tan β: posterior mean (best estimate) lies someway below benchmark

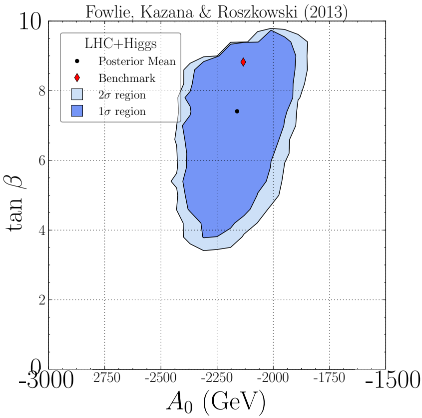
(A_0,tan β) for $\mathcal{L}_{\text{golden decay}}$



Adding information — Higgs mass — (A_0,tan β)

- ■/■ = 68%/95% region
  ♦/• = Benchmark/estimate
- Now add Higgs mass so that $\mathcal{L} = \mathcal{L}_{\text{g.d.}} \times \mathcal{L}_{\text{Higgs}}$
- **How much will extra info help reconstruction?**
- Increases tan β to saturate tree-level $m_h = M_Z \cos 2\beta$
- A_0 small improvement
- Still bias for smaller tan β

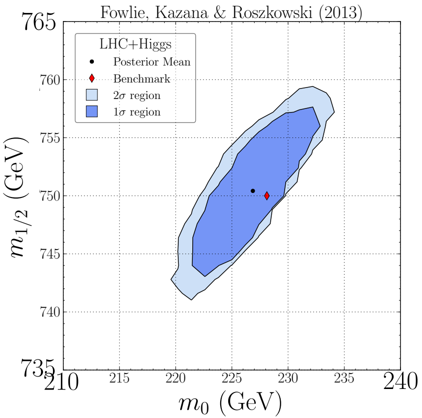
(A_0,tan β) for $\mathcal{L} = \mathcal{L}_{\text{g.d.}} \times \mathcal{L}_{\text{Higgs}}$



Adding information — Higgs mass — (m_0,$m_{1/2}$)

- Increase in $\tan\beta$ makes $\tilde{\tau} < \tilde{\chi}_1^0$
- Rules out left-hand-side (we want χ LSP)
- Otherwise little improvement in m_0 or $m_{1/2}$
- Higgs mass increases logarithmically with stop masses
- Insensitive to small changes in m_0 and $m_{1/2}$
- m_h cannot help once golden decay applied

(m_0,$m_{1/2}$) for $\mathcal{L} = \mathcal{L}_{\text{g.d.}} \times \mathcal{L}_{\text{Higgs}}$

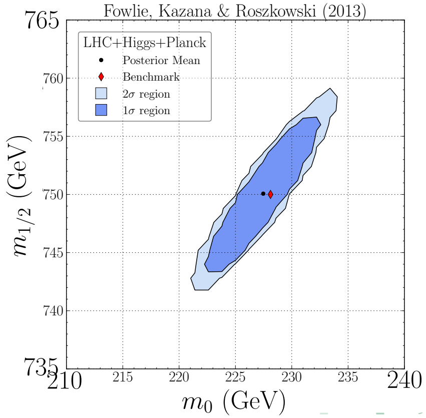


Results, all information, adding Ωh^2 — $(m_0, m_{1/2})$

- Add Planck measurement of Ωh^2
- $\mathcal{L} = \mathcal{L}_{\text{g.d.}} \times \mathcal{L}_{\text{Higgs}} \times \mathcal{L}_{\Omega h^2}$
- Despite all this information, picture not much improved
- Ωh^2 and golden decay constrain same direction of parameter space
- Major axis $\approx$ no improvement
- Minor axis squeezed by $\tilde{\tau} \approx \tilde{\chi}_1^0$ for stau-coannihilation
- Already determined by golden decay

$(m_0, m_{1/2})$ for

$\mathcal{L} = \mathcal{L}_{\text{g.d.}} \times \mathcal{L}_{\text{Higgs}} \times \mathcal{L}_{\Omega h^2}$

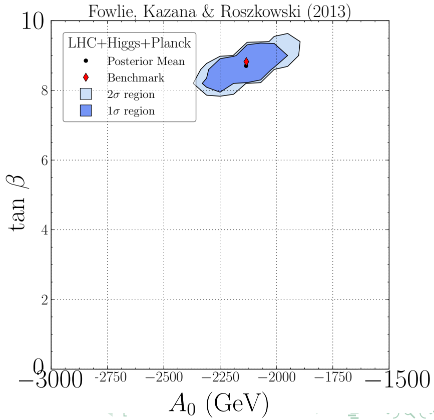


Results, all information, adding Ωh^2 — $(A_0, \tan\beta)$

- Big improvement in $\tan\beta$, now determined to within ~ 1 unit
- $\tan\beta$ tuned for $\tilde{\tau} \approx \tilde{\chi}_1^0$ stau-coannihilation
- Bias has disappeared
- Additional m_h and Ωh^2 information cannot help with A_0
- Determined still to within $\sim 0.5\,\text{TeV}$

$(A_0, \tan\beta)$ for

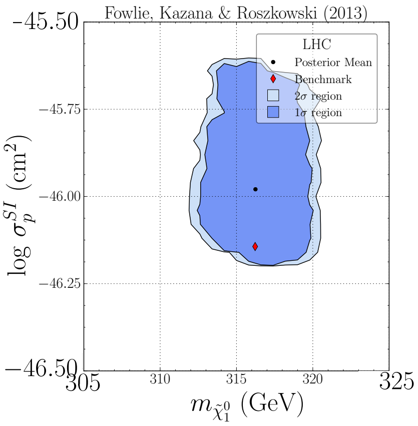
$\mathcal{L} = \mathcal{L}_{\text{golden decay}} \times \mathcal{L}_{\text{Higgs}} \times \mathcal{L}_{\Omega h^2}$



Information on direct detection

- σ_p^{SI} determined to within less than a decade
- Could know $\approx \sigma_p^{\mathrm{SI}}$ and whether neutralino was in reach of experiments
- Uncertainty in $\mathcal{B}\left(B_s \to \mu^+\mu^-\right)$, dominated by parametric Δm_t
- Δm_t more important than uncertainties in masses

$(m_\chi, \sigma_p^{\mathrm{SI}})$ for $\mathcal{L} = \mathcal{L}_{\text{golden decay}}$



Conclusions

- Simulated golden decay at high mass CMSSM benchmark point
- Found that sparticle masses can be measured with good precision
- Reconstructed CMSSM parameters with Bayesian statistics
- Found that CMSSM parameters can be well-recovered
- Except A_0, which is tricky
- Improves somewhat when additional information from Ωh^2 is added, but less so for m_h

Backup: Pythia

- $10\,\mathrm{k}$ events $\Rightarrow 85\,\mathrm{fb}^{-1}$
- $\sigma_{\mathrm{LO}} = 116.5\,\mathrm{fb}$
- Simplifying approximations:
  1. No detector effects
  2. No trigger
  3. Basic kinematic cuts, e.g., η within detector, p_T, E_T and j and ℓ
- Likelihood functions for Higgs and Ωh^2:
  1. $\mathcal{L}(h) = \exp\left[-\frac{(125.8\,\mathrm{GeV} - m_h)^2}{2((0.6\,\mathrm{GeV})^2 + (3\,\mathrm{GeV})^2)}\right]$
  2. $\mathcal{L}\left(\Omega h^2\right) = \exp\left[-\frac{\left(0.1186 - \Omega h^2\right)^2}{2(0.0031^2 + (0.1\Omega h^2)^2)}\right]$

Backup: Masses

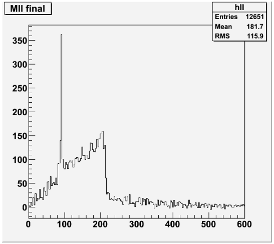
Particle	Mass (GeV)						
$\tilde{\chi}_1^0 = \chi$	316.2	$\tilde{e}_L$	553.5	$\tilde{d}_L$	154.6	h	123.2
$\tilde{\chi}_2^0$	603.6	$\tilde{e}_R$	364.7	$\tilde{d}_R$	147.9	H	1484.9
$\tilde{\chi}_3^0$	1394.0	$\tilde{\nu}_e$	547.8	$\tilde{u}_L$	154.5	A	1485.6
$\tilde{\chi}_4^0$	1397.9	$\tilde{\tau}_1$	318.3	$\tilde{u}_R$	148.5	$H^\pm$	1487.9
$\tilde{\chi}_1^\pm$	603.8	$\tilde{\tau}_2$	543.6	$\tilde{b}_1$	1277.9		
$\tilde{\chi}_2^\pm$	139.8	$\tilde{\nu}_\tau$	534.6	$\tilde{b}_2$	1463.6		
$\tilde{g}$	1675.1			$\tilde{t}_1$	821.5		
				$\tilde{t}_2$	1328.3		

Backup: Priors

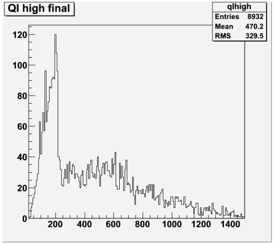
Parameter	Description	Prior range	Distribution
m_0	Unified scalar mass	(0.1, 4) TeV	Flat
$m_{1/2}$	Unified gaugino mass	(0.1, 2) TeV	Flat
A_0	Unified trilinear	(−4, 4) TeV	Flat
$\tan\beta$	Ratio of Higgs vevs	(3, 62)	Flat
$\mathrm{sgn}\,\mu$	Sign of Higgs parameter	$+1$	Fixed
m_t	Top pole mass	173.5 GeV	Fixed
$m_b(m_b)^{\overline{MS}}$	Bottom running mass	4.19 GeV	Fixed
$1/\alpha_{\mathrm{em}}(M_Z)^{\overline{MS}}$	Inverse of EM coupling	0.1184	Fixed
$\alpha_s(M_Z)^{\overline{MS}}$	Strong coupling	127.944	Fixed

Backup: Invariant mass distributions

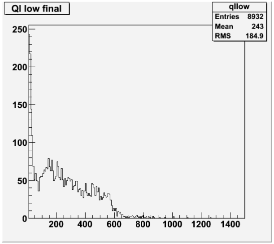
$\ell\ell$



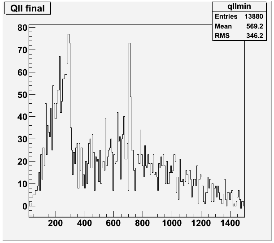
ℓq high



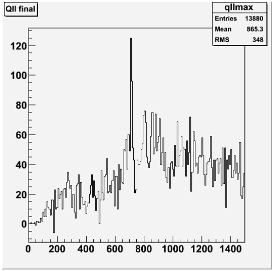
ℓq low



$q\ell\ell$ edge



$q\ell\ell$ threshold



Backup: Endpoints

- Left-handed slepton dominates golden decay
- Squark is average of squark masses
- Edges for our benchmark:
  1. $m^2_{\ell\ell} = 197.5\,\mathrm{GeV}$
  2. $m^2_{\ell q,\,\mathrm{high}} = 1052.1\,\mathrm{GeV}$
  3. $m^2_{\ell q,\,\mathrm{low}} = 511.0\,\mathrm{GeV}$
  4. $m^2_{\ell qq,\,\mathrm{edge}} = 1091.8\,\mathrm{GeV}$
  5. $m^2_{\ell qq,\,\mathrm{threshold}} = 380.3\,\mathrm{GeV}$

Backup: χ^2

- Observable predictions and χ^2:
  - $b \to s\gamma = 3.04 \times 10^{-4}$ $\chi^2 = 3.2$
  - $B_s \to \mu^+\mu^- = 3.18 \times 10^{-9}$ $\chi^2 = 0.0$
  - $B_u \to \tau\nu = 1.00$ $\chi^2 = 0.0$
  - $\Delta M_{B_s} = 21.35\,\mathrm{ps}^{-1}$ $\chi^2 = 2.2$
  - $\delta a_\mu = 3.87 \times 10^{-10}$ $\chi^2 = 9.5$
  - $h = 123.15\,\mathrm{GeV}$ $\chi^2 = 0.7$
  - $m_t = 175.0\,\mathrm{GeV}$ $\chi^2 = 2.3$
  - $M_W = 80.38\,\mathrm{GeV}$ $\chi^2 = 0.4$
  - $\Omega h^2 = 0.11$ $\chi^2 = 0.2$
  - $\sigma_p^{SI} = 7.19 \times 10^{-11}\mathrm{cm}^2$ $\chi^2 = 0$
  - $\sin\theta_{\mathrm{eff}} = 0.2314$ $\chi^2 = 1.5$
- With δa_μ, total $\chi^2 = 19.2$ with 11 degrees of freedom
- p-value $\gtrsim 5\%$
- Without δa_μ, total $\chi^2 = 9.7$ with 10 degrees of freedom
- p-value $\approx 50\%$