Origins of parameters in adimensional models

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Renormalization & QFT

Renormalization

Nutshell

- Physical theory with some parameters λ — describing some phenomena
- "Conditions" — temperature, electric field, magnetic field, pressure — change
- Do we need a new theory?
- Not necessarily. Renormalize parameters λ — adjust them but keep same theory
- For example, if changing temperature, T, promote $\lambda \to \lambda(T)$

Ball in water

- Consider a ball of mass m and volume V
- Acceleration follows Newton's second law

$$F = ma$$

- Now put the ball under water — fluid of density ρ
- Does Newton's second law hold?

Ball in water

- No! To accelerate the ball, you need to move the ball and the water in front of it
- However, we can renormalize the mass

$$m_R = m + \frac{1}{2} V \rho$$

- Newton's second law holds in the form

$$F = m_R \, a$$

- The renormalized mass is a function of density, $m_R = m_R(\rho)$

Lessons from PHY002 – dielectrics

Once you see renormalization, it's everywhere

- Capacitance of a paralell plate capacitor in vacuum

$$C = \epsilon_0 \frac{A}{d}$$

- Capacitance of a paralell plate capacitor with a dielectric

$$C = \epsilon \frac{A}{d}$$

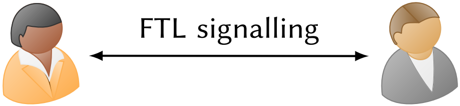
Quantum Field Theory

Nutshell

- Quantum Field Theory (QFT) — theory of fundamental particles
- Governs how particles behave — particle masses and interaction strengths
- Combines special relativity and quantum mechanics
- Experimentally tested to extraordinary precision

Why fields?

- In QM meausing observable 2 may impact observable 1 — observables needn't commute, $[O_1, O_2] \neq 0$
- What if light wouldn't have enough time to travel from measurement 1 to measurement 2 — measurements are space-like separated
- Faster-than-light signalling!



Alice measures O_1

Bob re-measures O_2; may learn about Alice's measurement

Why fields?

- Observables must be attached to space-time points
- If light wouldn't have enough time to travel between measurements, we make sure the measurements cannot impact each other
- Technically, observables in the theory must satisfy

$$[O_1(x), O_2(y)] = 0 \text{ if } (x - y)^2 < 0$$

- Thus we need to attach observables to space-time points — thus, we use fields, $\phi(x)$
- Fields are forced on us by combining QM and special relativity

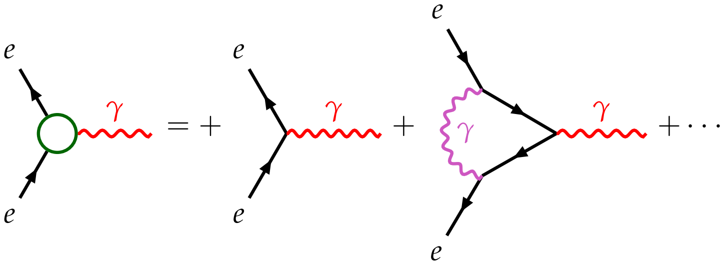
Renormalization in QFT

Nutshell

- Just like the ball in water
- The way particles interact depends on the characteristic energy of the interaction
- If you want to predict physics at a different energy, don't throw out the theory
- Keep the theory, but renormalize the parameters as functions of energy

Renormalization group equations

- This theory is relativistic $-$ $E = mc^2$ means that particles can be created from energy
- This impacts the interaction between, e.g., an electron and a photon



- In the tree-level diagram, the electron and photon field interact with strength α
- There are, though, heaps of corrections

Renormalization group equations

- As we change energy, loops of particles may become more relevant
- We can change α to incorporate the impact of the loops
- Parameter dependence on energy governed by differential equations — reormalization group equations — e.g.,

$$\frac{d\alpha}{d \ln Q} = \beta_0 \alpha^2 + \cdots$$

- The coupling is said to run with energy

Adimensional models

Hierarchy problem

- Scalar fields — trivial representation of Lorentz group — aren't protected from quantum corrections
- Their masses receive enormous quantum corrections, e.g., from quantum gravity

$$m^2 = m_0^2 + M_{\text{Pl}}^2$$

- The Planck mass, $M_{\text{Pl}} \approx 10^{19}\,\text{GeV} \approx 10^{-8}\,\text{kg}$, is scale at which gravity similar in strength to other forces

$$M_{\text{Pl}} = \sqrt{\frac{\hbar c}{G}}$$

- Planck mass similar to mass of speck of flour!
- The unit $1\,\text{GeV} \simeq 10^{-27}\,\text{kg}$ similar to mass of proton

Hierarchy problem

In a nutshell

- We observe that scale of elementary particle physics

$$m^2 \approx (100\,\mathrm{GeV})^2 \lll M_{\mathrm{Pl}}^2$$

- Fine-tuning between m_0^2 and M_{Pl}^2 — require

$$m_0^2 = -(10^{19}\,\mathrm{GeV})^2 + (100\,\mathrm{GeV})^2$$

- Ugly, unnatural and moreover implausible

$$m_0^2 = -99\,999\,999\,999\,999\,999\,999\,999\,999\,999\,999\,990\,000 \text{ GeV}^2$$

Solutions

- Since the 1980s, model-building in high-energy physics focussed on solving the hierachy problem
- In other words, building theories that predicted $m^2 \lll M_{\mathrm{Pl}}^2$ and didn't need fine-tuning
- All attempts to do so introduce new particles with masses just above 100 GeV
- New physics that could be observed in particle colliders
- Most popular models were supersymmetry (SUSY), including supersymmetric grand unified theories (GUTs)

LHC lessons

In a nutshell

- Hang on. Have you seen the LHC results?
- No supersymmetry. No large extra dimensions. No signatures of any new particles near 100 GeV
- Maybe there are no new particles (Foot et al., 2008; Heikinheimo et al., 2014; Gabrielli et al., 2014; Englert et al., 2013; Kannike, Racioppi, and Raidal, 2014)
- The top quark is the heaviest known particle, $m_t \simeq 170\,\mathrm{GeV}$
- Maybe the top really is the top

Traditional picture

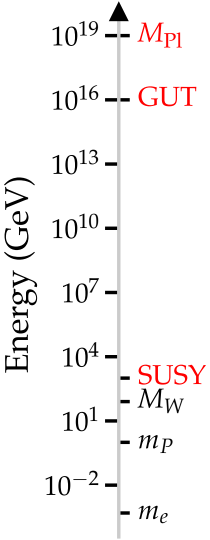


Figure: Succession of massive scales and new physics. Hierarchy problems.

Adimensional picture

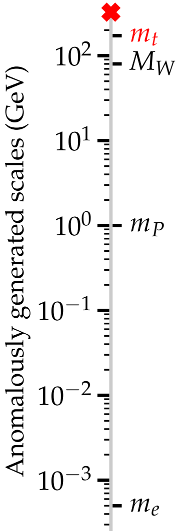


Figure: The top is the top. No bigger scales. No hierarchy problems.

Adimensional models

- No fundamental dimensional constants in nature (Salvio and Strumia, 2014)
- No exotic physics or strings — just QFT
- All scales generated by quantum effects
- For example, consider Newtonian gravity between two masses

$$F = G\frac{m_1 m_2}{r^2}$$
$$G = 6.67 \times 10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$$

- Newton's constant G is dimensional
- Must be generated by quantum effects

Adimensional models

- There are massive scales in Nature but they cannot appear explicitly in the Lagrangian
- This is the phenomena of dimensional transmutation — could be Coleman-Weinberg mechanism or confinement
- No massive corrections to scalar masses

$$\Delta m^2 = \cancel{M_{\text{Pl}}^2} = 0$$

  on dimensional grounds — nothing to put on the right-hand side (Bardeen, 1995)
- No hierarchy problem

Can that really work?

Can that really work?

What about gravity?

- Gravity governed by dimensional parameter, $G \sim 1/M_{\mathrm{Pl}}^2$
- However, quadratic gravity (Salvio, 2018; Donoghue and Menezes, 2022) is adimensional and renormalizable
- Suffers from Ostrogradsky instability or ghosts, though may be viable (Salvio and Strumia, 2016; Raidal and Veermäe, 2017; Strumia, 2019; Gross et al., 2021; Donoghue and Menezes, 2021)
- No easy path to quantum gravity; must consider paths with obstacles

Origins of fundamental parameters

- Deep question — what is the origin of our fundamental parameters?
- Adimensional models posit no new dimensional physics
- Nothing exotic or dramatic at high energies — no strings etc, just QFT
- I don't know any QFT that can explain its fundamental parameters
- Does it close door for explanations of fundamental parameters?

Profound consequences

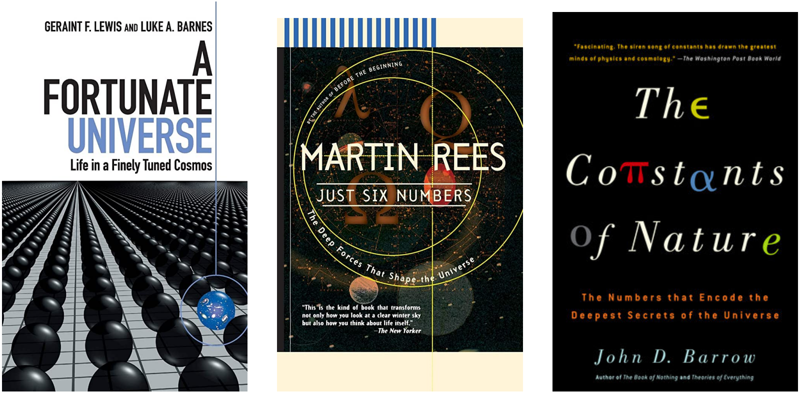


Figure: Explored in popular science but rarely in research

Are they random?

- Perhaps fundamental parameters are randomly chosen
- From what distribution?
- If there are no fundamental scales, that distribution had better not depend on any scale
- We must find distributions for an adimensional theory's dimensionless parameters that don't refer to any particular dimensional scales

Are they random?

- If there are fundamental scales, the distribtuions could be scale dependent
- In grand unified theories (GUTs), for example, we might assume there is a special unification scale, M_X
- We could write distributions for a unified coupling at that scale, $Q \approx M_X$
- They would look different at other scales, but that's alright as M_X is special

Invariant measures

Changing variables

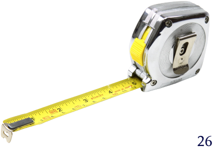
- The densities for a parameter $y = f(x)$ and x are connected by

$$p_Y(y) = p_X(x)\ |\mathcal{J}|$$

  where $|\mathcal{J}|$ is the Jacobian for the transformation between x and y

- In this simple one-dimensional case,

$$|\mathcal{J}| = \left|\frac{dx}{dy}\right|$$

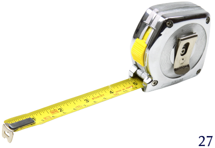


Invariant measures

- The distribution would be invariant under this transformation if p_X and p_Y were the same function,

$$p \equiv p_Y = p_X$$

- Formally, $p(x)dx$ would be an invariant measure (see e.g., Hartigan, 1964; Jaynes, 1968; Dawid, 2006; Consonni et al., 2018)

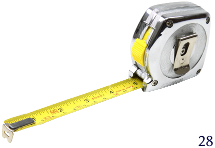


Measure theoretic

- Measure — assigns non-negative value to subsets of a space
- Must satisify

$$\mu(\phi) = 0 \quad \text{and} \quad \mu(\cup_i A_i) = \sum_i \mu(A_i)$$

  for disjoint sets A_i
- Further conditions for a probability measure, e.g., probability on A satisfies $\mu(A) = 1$ and A is a σ-algebra



Topological group

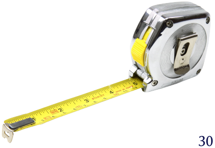
- The parameter transformations may form a topological group
- Requires that transformations are continuous, associative and closed, and the existence of an inverse and an identity
- The topological space may be locally compact — no holes or spikes — in which case the group is said to be locally compact
- The real numbers — topological group under addition and locally compact as no holes
- The group parameters may be defined on a closed interval, in which case the group is said to be compact
- Lorentz group — defined on $[0, c)$ — not compact

Haar measure

- A (right) invariant measure satisfies

$$\mu(S) = \mu(Sg)$$

  for every subset S and every group element g
- This invariant measure is the (right) Haar measure of the group
- Haar measure natural notion of volume



Existence of Haar measure

- Haar measure exists for any locally compact group
- Though proper — $\mu(G) < \infty$ — if and only if group is compact

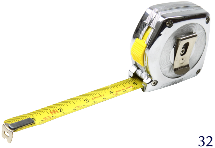
Shifting

- Consider the reals and the transformation $x \to x + A$
- The invariant measure is the Lebesgue measure

$$\mu([x, y]) = x - y$$

- The invariant distribution $p(x)dx \propto \mu(dx)$ is simply

$$p(x) \propto \text{const.}$$



Scaling

- Consider the reals and the transformation $x \to Ax$
- The invariant measure is more involved

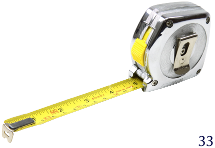
$$\mu([x, y]) = \log y - \log x$$

- The invariant distribution is

$$p(x) \propto \frac{1}{x}$$

- or equivalently,

$$p(\log x) \propto \text{const.}$$



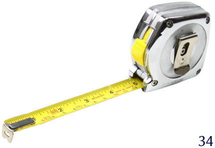
Impropriety

- The scale and shift invariant distributions are examples of improper distributions
- They weren't compact spaces
- They cannot be normalised to one because

$$\int p(x)dx = \infty$$

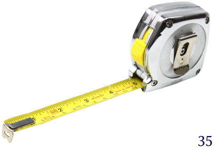
- We cannot sample from them
- Not useless, however, as improper prior + likelihood may lead to a proper posterior

$$\int p(K \mid x)p(x)dx < \infty$$



Invariants

- The number of distributions that can be found by a group invariance is equal to the size of the invariance group, n
- We can re-parameterise our d dimensional space as n parameters and $d - n$ group invariants
- For example, rotations in d dimensions — an invariant radius and $d - 1$ angles
- The measures for the invariants are arbitrary as they do not transform under the group
- $|\mathcal{J}| = 1$ for the invariants



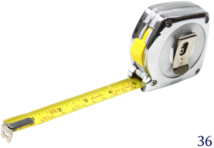
Examples

- Suppose that we considered dependent re-scalings for two parameters, $x \to Ax$ and $y \to Ay$
- Invariance cannot uniquely dictate the form of the two-dimensional prior, as it could be

$$p(x,y) \propto \frac{f(x/y)}{xy}$$

- x/y is a group invariant
- The function f isn't restricted, though it must satisfy

$$\int f(z)dz = 1$$



Examples

- Similarly, if we were to consider dependent shifts, $x \to x + A$ and $y \to y + A$, we would obtain

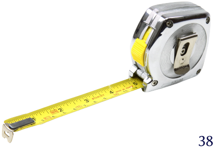
$$p(x, y) \propto f(x - y)$$

- $x - y$ is a group invariant
- The function f isn't restricted, though it must satisfy

$$\int f(z) dz = 1$$

RG invariant distributions

- We want to construct distributions for an adimensional theory's parameters that don't refer to any particular dimensional scale
- We must consider the RG evolution of the parameters and build the RG invariant measures and distributions
- If we succeed, fundamental parameters could be draws from these distributions
- If we fail, the parameters must originate in some other way



Examples

QCD β-function

- The β-function in QCD

$$\frac{d\alpha_s}{d\ln Q} = -\beta_0 \alpha_s^2 \quad \text{where} \quad \beta_0 > 0$$

- Solving this differential equation yields

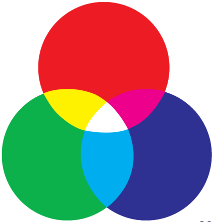
$$\alpha_s(Q) = \frac{\alpha_s(Q')}{1 - \alpha_s(Q')\,\beta_0 \ln\left(Q'/Q\right)}$$

- This may be re-written as

$$\alpha_s(Q) = \frac{1}{\beta_0 \ln\left(Q/\Lambda_{\mathrm{QCD}}\right)}$$

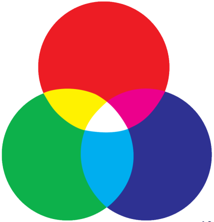
- This used the RG-invariant QCD scale

$$\Lambda_{\mathrm{QCD}} = Q e^{-\frac{1}{\beta_0 \alpha_s(Q)}}$$



QCD RG flow

- The coupling flows to $\alpha_S = 0$ — asymptotic freedom
- This is a UV attractor
- Landau-pole — finite-time blow up — in the IR at the QCD scale
- RG evolution through Landau pole impossible



QCD RG flow

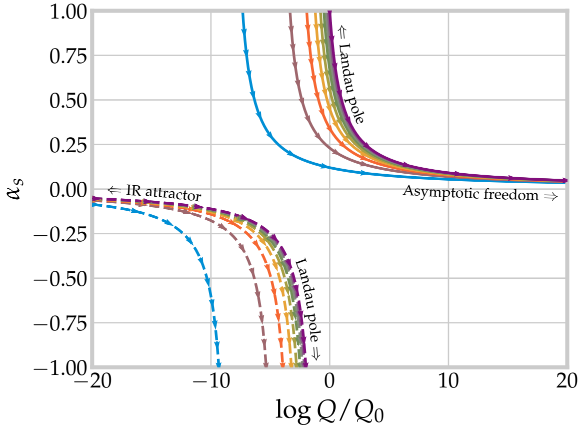


Figure: RG flow in QCD.

QCD RG flow

- We cannot evolve through Landau pole
- The RG transformations aren't closed — as evolving to $Q < \Lambda_{\text{QCD}}$ undefined
- This means that an invariant measure cannot exist
- For illustrative purposes, suppose

$$\alpha_s(Q) = \frac{1}{\beta_0 \ln\left(Q/\Lambda_{\text{QCD}}\right)}$$

valid for any Q

- The Jacobian for flowing from $Q \to Q'$

$$|\mathcal{J}| = \left|\frac{d\alpha_s(Q')}{d\alpha_s(Q)}\right| = \left|\frac{\alpha_s(Q')}{\alpha_s(Q)}\right|^2$$

Invariant measure

- The Jacobian rule leads to

$$p(\alpha_s(Q)) = p'(\alpha_s(Q'))\,|\mathcal{J}|$$

- Requiring that $p = p'$ gives an invariant distribution

$$p(\alpha_s) \propto \frac{1}{\alpha_s^2}$$

- Wonderful! but improper!

$$\int_0 \frac{d\alpha}{\alpha^2} = \infty$$

Why is it improper, physically?

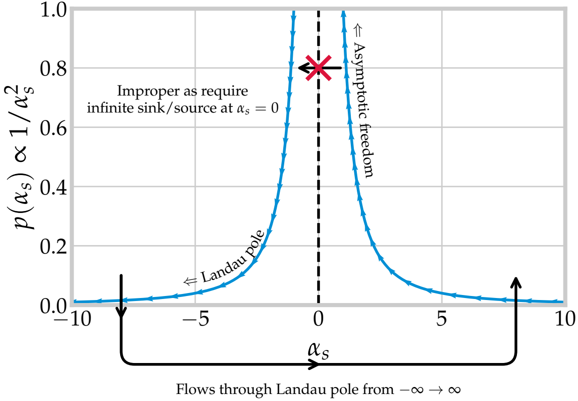


Figure: We cannot cross from $+0$ to -0

Why is it improper, physically?

- The invariant measure for the QCD coupling leads to

$$p(\log \Lambda_{\mathrm{QCD}}) = \text{const.}$$

- This is a scale invariant distribution — invariant under

$$\Lambda_{\mathrm{QCD}} \to A\Lambda_{\mathrm{QCD}}$$

- Intuitive — how could there be any preferred QCD scale in the theory?
- Improper — having no preferred scale means that it is improper

Why is it improper, group theoretically?

- The group wasn't compact; specifically, as we omitted $\alpha = 0$
- $\alpha = 0$ trivial; stays zero forever
- If included, the invariant measure is a Dirac mass at zero, $\delta(\alpha)$

Does it matter?

- In our physics education, we have a series of shocks when we discover seeming important quantities can be set arbitrarily, e.g.,

$$c = \hbar = G = 1$$

- We can use Λ_{QCD} to define a system of units

$$\Lambda_{\mathrm{QCD}} = 1$$

- Measure any other scales relative to QCD scale

Would this work in realistic models?

- In principle, more realistic models are no different
- If RG flows forms locally compact group, invariant distribution should exist
- Though only proper if flow forms compact group
- In practice, much harder as we cannot compute the RG invariants or solve the RG equations analytically
- Maybe there are short-cuts?
- For example, Ulam's method or the ergodic hypothesis that time-averages equal space-averages

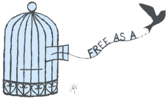
Conclusions

- Adimensional models could solve the hierarchy problem
- Possibly leads to renormalizable quadratic gravity, though these theories are pathological even at the classical level
- Predict no fundamental scales at all
- If parameters originate as random draws, distributions must be scale invariant
- Explored scale invariant distributions in simple models
- This involves finding the invariant Haar measure of RG transformations
- Finite-time blow-up (Landau poles) cause difficulties

Backup

Total Asymptotic Freedom

- QCD was trivial — $\Lambda_{\mathrm{QCD}} = 1$
- Scalar QED suffered from Landau poles in the IR and UV
- Consider theory with no Landau poles in UV — easiest theories are totally asymptotically free theories (Giudice et al., 2015)
- Make a simple model — a scalar with a non-Abelian gauge interaction
- Leave it general — don't specify group or particle content



Total Asymptotic Freedom

- Form of RG equation for the gauge coupling identical to that in QCD — no Landau pole in UV
- Agnostic about form of quartic RG equation;

$$\frac{d\lambda}{d \ln Q} = s_\lambda \lambda^2 - s_{\lambda g} \lambda g^2 + s_g g^4$$

- In known QFT, coefficients $s > 0$

RG flow in simple TAF model

- Convenient to define

$$C \equiv \frac{s_g}{\beta_0}$$

$$D \equiv \frac{s_{\lambda g} - \beta_0}{2 s_g}$$

$$E \equiv D^2 - \frac{s_\lambda}{s_g}$$

- For $E = 0$, trivial fixed-flow
- For $E < 0$, Coleman-Weinberg type behaviour — tangent that swings rapidly between Landau poles
- For $E > 0$, may avoid Landau poles

RG flow in simple TAF model

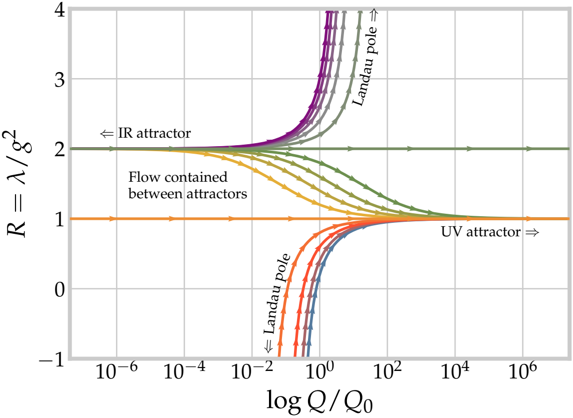


Figure: Flow contained between attractors

RG flow in simple TAF model

- The attractors are at

$$R_{\mathrm{IR}} = \frac{1}{D - \sqrt{E}} \quad \text{and} \quad R_{\mathrm{UV}} = \frac{1}{D + \sqrt{E}}.$$

- If at any scale the ratio lies inside $[R_{\mathrm{UV}}, R_{\mathrm{IR}}]$, it stays trapped inside that interval

$$\begin{aligned} R(Q) &\equiv \frac{\lambda(Q)}{4\pi\alpha(Q)} \\ &= R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}}) \frac{1}{2} \left[1 - \tanh\left(C\sqrt{E} \ln \alpha(Q) + \Theta \right) \right] \end{aligned}$$

- $R = R_{\mathrm{UV}}$ and $R = R_{\mathrm{IR}}$ are special fixed flows
- Otherwise, it flows to a Landau pole in the IR or UV

RG flow in simple TAF model

- Inside the solution

$$R(Q) = R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}})\,\frac{1}{2}\left[1 - \tanh\left(C\sqrt{E}\ln\alpha(Q) + \Theta\right)\right]$$

  the red factor goes from 0 in the IR to 1 in the UV

- Θ is an RG invariant,

$$\Theta = \operatorname{arctanh}\left[1 - 2\left(\frac{R(Q) - R_{\mathrm{IR}}}{R_{\mathrm{UV}} - R_{\mathrm{IR}}}\right)\right] - C\sqrt{E}\log\alpha(Q)$$

- It controls $R(Q')$

RG flow in simple TAF model

- Computing the Jacobian, we find an RG invariant measure on $(R_{\mathrm{UV}}, R_{\mathrm{IR}})$,

$$p(R \mid \alpha) \propto \frac{f(\Theta)}{(R_{\mathrm{IR}} - R)\,(R - R_{\mathrm{UV}})}$$
$$p(\alpha) \propto \frac{1}{\alpha^2}$$

- Conditional distribution $p(R \mid \alpha)$ has poles at attractors
- Proper so long as $f(\Theta)$ is proper
- Same form at every scale; though shape of distributions flows as α flows

Role of $f(\Theta)$

- $\Theta \equiv \Theta(R, \alpha)$ — invariant though function of R and α
- The function $f(\Theta)$ must satisfy

$$\int f(\Theta) d\Theta = 1$$

and thus must satisfy

$$\lim_{|\Theta| \to \infty} f(\Theta) = 0$$

- Consider behaviour of

$$R(Q) = R_{\mathrm{IR}} + (R_{\mathrm{UV}} - R_{\mathrm{IR}}) \frac{1}{2} \left[1 - \tanh\left(C\sqrt{E} \ln \alpha(Q) + \Theta\right)\right]$$

- In the IR where $\ln \alpha \to \infty$, $R \to R_{\mathrm{UV}}$ requires $\Theta \to -\infty$
- In the UV where $\ln \alpha \to -\infty$, $R \to R_{\mathrm{IR}}$ requires $\Theta \to \infty$

Role of $f(\Theta)$

Controls flow

- Thus the function $f(\Theta)$ must disfavour R_{UV} in the IR and R_{IR} in the UV
- Controls flow of probability from R_{IR} in the IR to R_{UV} in the UV

Now, as an example, consider a standard normal, $f(\Theta) = \mathcal{N}(0, 1)$ with $R_{\mathrm{IR}} = 2$ and $R_{\mathrm{UV}} = 1$

Measure flows between IR and UV attractor

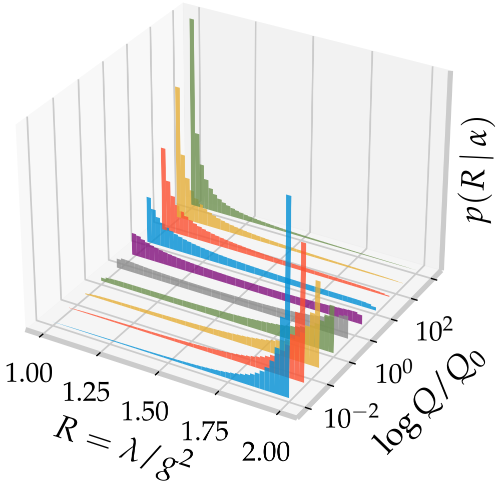


Figure: The measure moves the probability mass between the attractors

Dirac masses at attractors

- We considered $(R_{\mathsf{IR}}, R_{\mathsf{UV}})$, i.e., omitting the endpoints
- Dirac mass at the attractors would also be invariant

$$p(R) = \delta(R - R_{\mathsf{IR/UV}})$$

- They could be combined with our invariant distribution

What theories predict $E > 0$?

- Discussed in ref. (Giudice et al., 2015)
- Certainly possible, though requires big representations and big groups
- Easier if add Yukawa interactions, though RG equations become harder to solve

References I

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